

KU Mathematics and Statistics Competition
Department of Mathematics
University of Kansas



Competition for 6th-8th Grades
April 10, 2021

INSTRUCTIONS:

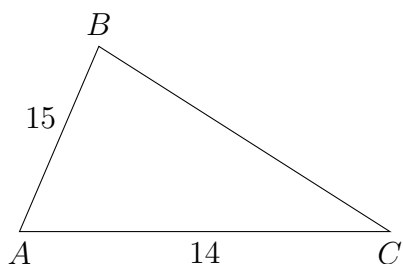
- You have **40 minutes** for the **five problems**.
 - Show all of the necessary work and **provide a complete justification for each answer**.
 - Solve **each problem on a separate sheet of paper**.
 - Enclose each **final answer in a box**.
 - You are allowed to use a calculator.
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Problem 1. The area of the scalene triangle below is 84 sq. units. Two side lengths are given as $AB = 15$ and $AC = 14$. Determine the length of the third side BC .

Note: *Figure below is not drawn to scale.*



Problem 2. We say that a whole number n is *even* if there exists a whole number k such that $n = 2k$, or we say that n is *odd* if there exists a whole number ℓ such that $n = 2\ell + 1$. We refer to the even- or oddness of n as its *parity*. Show first that the parity of a whole number n is the same as the parity of n^2 ; then, show that the parity of n is the same as the parity of n^3 . Ultimately, deduce that the parity of a whole number is the same as any power of itself.

Problem 3. In July 2018, the state of Kansas announced that its license plates would undertake a new look but the license plate number template would remain the same: number-number-number letter-letter-letter where each number ranges from 0 to 9 and each capitalized letter is any of the 26 letters in the alphabet, e.g., 123 KAN. Compute the probability that a Kansas license plate has two "3"s and one "C".

Problem 4. The smallest positive integer n with the property that 3 divides n , 4 divides $n + 1$, 5 divides $n + 2$, 6 divides $n + 3$, 7 divides $n + 4$, 8 divides $n + 5$, and 9 divides $n + 6$ is 3. What is the next large integer with this property?

Problem 5. Danily, Paszka, Satheryn and Terryyn go out for dinner. When it is time to pay, Satheryn realizes she (conveniently) forgot her wallet. In order to pay for Satheryn's meal, Danily contributes one fifth of her money, Paszka gives one quarter of her money, and Terryyn provides one third of her money. Each of them contributes the same amount of money and the sum of all contributions is exactly equal to the cost of Satheryn's meal. What fraction of the group's money is Satheryn's share of the total bill?