

Math Awareness Month Competition
University of Kansas, Department of Mathematics
2018 Examination for 9th-12th Grades

DIRECTIONS: You have 40 minutes for the five problems.

Show all of the necessary work and provide a complete justification for each answer.

Enclose each answer in a box. Solve each problem on a separate sheet of paper.

You are allowed to use a calculator but you are not allowed to borrow or interchange calculators during the test.

1. A wooden cube with edge length n inches (where $n > 2$ is an integer) is painted blue all over. By slices parallel to its faces, the cube is cut into n^3 smaller cubes each of an inch edge length. If the number of smaller cubes with just one face painted blue is equal to the number of smaller cubes completely free of paint, what is n ?
2. The probability that the event A occurs is $3/4$, the probability that the event B occurs is $2/3$. The events are not necessarily independent. Let p be the probability that both A and B occur. What is the smallest interval necessarily containing p ?
3. Exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon?
4. How many different values of n are there such that n is a natural number and $n^2 - 440$ is a perfect square?
5. Find the minimum value of

$$f(x) = \frac{9x^2 \sin^2 x + 4}{x \sin x}$$

for $0 < x < \pi$.