Math Awareness Month Competition 2010

Solutions for 5th-6th Grades

1. How many two-digit odd numbers can be made using the digits 0, 1, 2, 3, 4, 5, 7, 8 if each digit can be used at most once in a number?

[Solution: The second digit can be only 1, 3, 5 or 7. The first digit cannot be 0. If the first digit is even, then for each odd second digit we can choose the first of the digit in 4 ways. This is $3 \cdot 4 = 12$ possibilities. If the first digit is odd, then for each odd second digit we can choose any odd first digit but the second digit. This is $3 \cdot 4 = 12$ possibilities. Hence, the answer is 12+12= **24**.]

2. If 12 cubic yards of wood are burnt in 9 fireplaces in 16 days, then in how many days are 9 cubic yards of wood burnt in 12 fireplaces?

[Solution: 9 cubic yards of wood are burnt in 9 fireplaces in $\frac{9}{12} \cdot 16$ days. 9 cubic yards of wood are burnt in 12 fireplaces in $\frac{9}{12} \cdot \frac{9}{12} \cdot 16 =$ **9 days**.]

3. There are five sheds located on a farm. The distance between shed A and B is 2 miles, between B and C is 1.5 miles, between C and D is 8 miles and between D and E is 3.5 miles. Finally shed A is 1 mile from shed E. How far is shed B from shed E? Explain why this is the only possible solution.

[Solution: Among the given distances, the largest one equals the sum of all the others: $8 = 2 + 1.5 + 3.5 + 1$. This is possible only if the sheds are along a line. The order of the sheds must be C, B, A, E, D. Therefore, $\text{distant}(B, E) = \text{distant}(B, A) + \text{distant}(A, E) = 2 + 1 =$ **3 miles**.]

4. We write the numbers $1, 2, 3, 4, 5$ in any order, then we add the differences (with no signs) of the numbers next to each other. For example, in the order 21435, the sum = (difference of 21) + (difference of 14) + (difference of 43) + (difference of 35) = $1 + 3 + 1 + 2 = 7$. What is the largest possible value of this difference sum?

[Solution: The largest difference is $5 - 1 = 4$. Thus, 5 and 1 are next to each other. The second largest differences are $4 - 1 = 3$ and $5 - 2 = 3$. Thus, 4 and 1 are next to each other and similarly 5 and 2. Hence, a part of the number with the largest difference must contain either 4152 or 2514. This has a difference sum of 10 so far. Adding the remaining 3 to either end of the two numbers increases the difference sum by 1. Therefore, the largest of the difference sum is $\mathbf{11}$.]

5. On each face of a cube there is a different number from 1 to 6. If one of the faces is at the bottom of the cube, then the sum of the numbers on all the faces except the top and bottom is 13. But if another face is at the bottom, then the sum of the numbers on all the faces except the top and bottom is 12. What is the number written on the opposite side of the number 1?

[Solution: The sum of the numbers on all the faces is 21. The first observation implies that the sum of the numbers on a couple of opposite sides is 8. This couple can be $(6, 2)$ or $(5, 3)$. The second observation implies that the sum of the numbers on another couple of opposite sides is 9. That couple can be $(6, 3)$ or $(5, 4)$. Combining the above information, the only possibility is that $(6, 2)$ and $(5, 4)$ are on opposite sides. Hence, $(3, 1)$ is the third couple of opposite sides, therefore the answer is **3**.]