1. set $u = (\ln|x|)^2$, $dv = x\,dx$

$du = 2(\ln|x|)\frac{1}{x}$, $v = \frac{x^2}{2}$

$$\int x(\ln|x|)^2\,dx = \frac{1}{2}x^2(\ln|x|)^2 - \int x\ln|x|\,dx$$

set again $u = \ln|x|$, $dv = x\,dx$

$du = \frac{1}{x}dx$, $v = \frac{x^2}{2}$

$$\int x(\ln|x|)^2\,dx = \frac{1}{2}x^2(\ln|x|)^2 - \frac{1}{2}x^2\ln|x| + \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}x^2(\ln|x|)^2 - \frac{1}{2}x^2\ln|x| + \frac{1}{4}x^2 + C$$

2. set $y = \sin x$, $dy = \cos x\,dx$

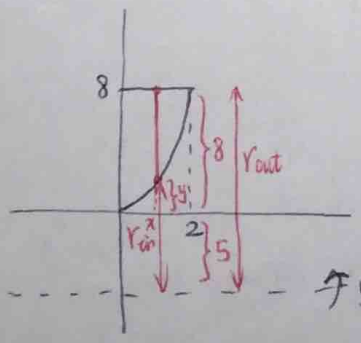
$$\int \cos x \ln|\sin x|\,dx = \int \ln|y|\,dy$$

$dv = dy$, $u = \ln|y|$, $du = \frac{1}{y}dy$, $v = y$

$$\int \cos x \ln|\sin x|\,dx = y\ln|y| - \int dy = y\ln|y| - y = \sin x \ln|\sin x| - \sin x + C$$

3. (a), (b) see Lab 4 solutions for Examples.

(c)



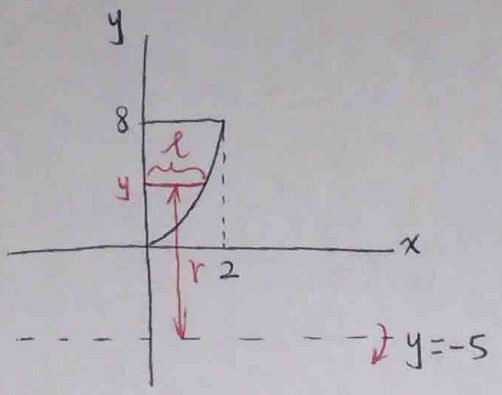
"washer":

$r_{out} = 8 - (-5) = 13$

$r_{in} = y - (-5) = y + 5 = x^3 + 5$

$$\int_0^2 \pi\left[13^2 - (x^3+5)^2\right]dx$$

#.



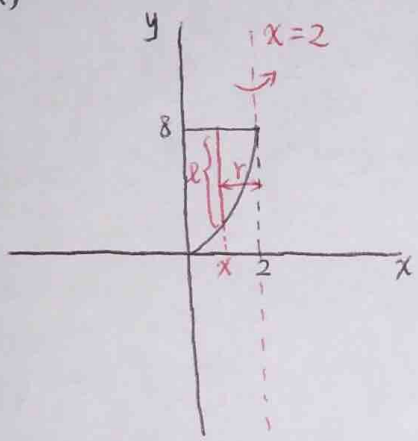
"shell"

$r = y + 5$

$\ell = x = y^{\frac{1}{3}}$

$$\int_0^8 2\pi(y+5)\,y^{\frac{1}{3}}\,dy$$

(d)

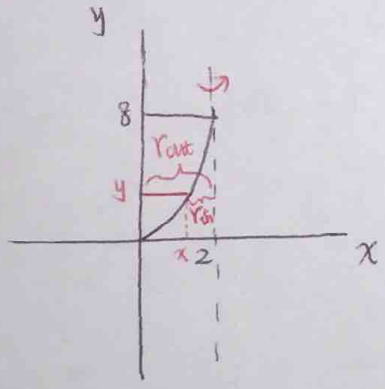


"shell"

$r = 2 - x$

$\ell = 8 - y = 8 - x^3$

$$\int_0^2 2\pi(2-x)(8-x^3)\,dx$$



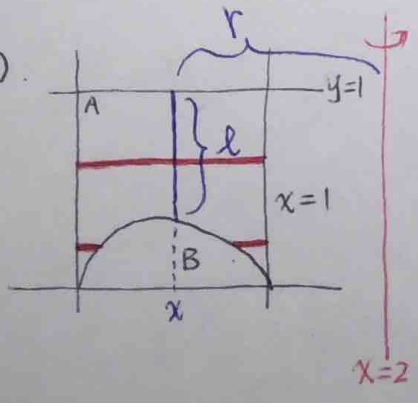
"Washer"

$r_{out} = 2$

$r_{in} = 2 - x = 2 - y^{\frac{1}{3}}$

$$\int_0^8 \pi\left[2^2 - (2 - y^{\frac{1}{3}})^2\right] dy$$

5. (a).



Red sticks change "walls", so washer method is not a good idea to apply here.

Apply "shell" method: $r = 2 - x$,

$\ell = 1 - y = 1 - (x - x^4)$

$$\int_0^1 2\pi(2-x)(1-x+x^4)\,dx$$