Integration Problems

1. Evaluate the following definite and indefinite integrals:

(a) $\displaystyle\int \frac{dx}{(x-5)^9}$

(b) $\displaystyle\int (9t+2)^{\frac{2}{3}}$

(c) $\displaystyle\int \frac{5x^4+2x}{(x^5+x^2)^3}\,dx$

(d) $\displaystyle\int \frac{x^2}{(x+5)^3}\,dx$

(e) $\displaystyle\int x\sin(x^2)\,dx$

(f) $\displaystyle\int \frac{\tan(\ln(x))}{x}\,dx$

(g) $\displaystyle\int \frac{e^x}{(e^x+1)^4}\,dx$

(h) $\displaystyle\int \sin(45t)\sec^2(\cos(45t))\,dt$

(i) $\displaystyle\int \frac{\cos(\pi/x^{29})}{x^{30}}\,dx$

(ii) $\displaystyle\int \sqrt{x^5}\sin(2+x^{7/2})\,dx$

(iii) $\displaystyle\int \sec^2(\theta)\tan^7(\theta)\,d\theta$

(iv) $\displaystyle\int_9^{10} x\sqrt{x-9}\,dx$

(v) $\displaystyle\int_0^{31} \frac{dx}{\sqrt[3]{(1+4x)^2}}$

(vi) $\displaystyle\int_0^{\pi/10} \cos(5x)\sin(\sin(5x))\,dx$

(vii) $\displaystyle\int_0^{\sqrt[14]{\pi}} x^{13}\cos(x^{14})\,dx$

2. Todd uses the substitution $u=\tan(x)$ and Vanessa uses $v=\sec(x)$ to evaluate $\displaystyle\int \tan(x)\sec^2(x)\,dx$. Show that they obtain different answers and explain the contradiction.

3. Use substitution to evaluate the integral in terms of $f(x)$.

(a) $\displaystyle\int f(x)^3 f'(x)\,dx$

(b) $\displaystyle\int f'(-x+7)\,dx$

Section 6.1 Problems

1. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area.

(a) $y=2x+3$, $y=13-x^2$, $x=-1$, and $x=2$.

(b) $x=45-5y^2$ and $x=5y^2-45$.

(c) $y=6\cos(\pi x)$ and $y=12x^2-3$.

(d) $y=4\cos(6x)$ and $y=4\sin(12x)$ on the interval $\left[0,\frac{\pi}{12}\right]$.

(e) $y=\sqrt{x}$, $y=\frac{1}{2}x$, and $x=25$.

(f) $y=|3x|$ and $y=x^2-4$.

2. Find the number b such that the line $y=b$ divides the region bounded by the curves $y=4x^2$ and $y=16$ into two regions with equal area.

3. Sketch the region under the graph of $f(x)=\dfrac{1}{1+x^2}$ and then show that the total area contained between the curve and the x-axis is π units^2.