1. $\begin{vmatrix} -6 & 9 \\ 1 & 1 \end{vmatrix} = -6-9=-15$

$\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix} = -3-8+3=-8$

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -4 & 2 \end{vmatrix} = 0$

2. $\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ -2 & 5 & 10 \\ -6 & -3 & 5 \end{vmatrix} = \begin{vmatrix} 5 & 10 \\ -3 & 5 \end{vmatrix} i - \begin{vmatrix} -2 & 10 \\ -6 & 5 \end{vmatrix} j + \begin{vmatrix} -2 & 5 \\ -6 & -3 \end{vmatrix} k$

$= <55, -50, 36>$

$\vec{u}\cdot(\vec{u}\times\vec{v}) = <-2, 5, 10>\cdot<55, -50, 36> = -110-250+360=0$

so $\vec{u}\perp(\vec{u}\times\vec{v})$ Similarly one can check $\vec{v}\perp(\vec{u}\times\vec{v})$

3. $\vec{v}\times\vec{u} = -\vec{u}\times\vec{v} = -<1,1,0>$

$\vec{w}\times(\vec{u}+\vec{v}) = \vec{w}\times\vec{u}+\vec{w}\times\vec{v} = -\vec{u}\times\vec{w}-\vec{v}\times\vec{w}$

$= -<0,3,1> - <2,-1,1> = <-2,-2,-2>$

$(3\vec{u}+4\vec{w})\times\vec{w} = 3(\vec{u}\times\vec{w})+4(\vec{w}\times\vec{w})$

$= 3(\vec{u}\times\vec{w}) = 3<0,3,1>$

$(\vec{u}-2\vec{v})\times(\vec{u}+2\vec{v}) = \vec{u}\times\vec{u}+2(\vec{u}\times\vec{v})-2(\vec{v}\times\vec{u})-4(\vec{v}\times\vec{v})$

$= 4(\vec{u}\times\vec{v}) = 4<1,1,0>$

4. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} i - \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} j + \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} k$

$= <-1, -4, 7>$

$\vec{b}\times\vec{a} = -<-1,-4,7>$ both these two vectors $\perp\vec{a}$, $\perp\vec{b}$

5. $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = \sin\theta = \frac{1}{2}$, so $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

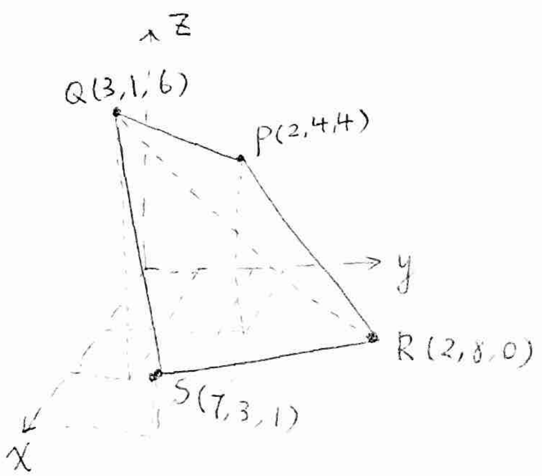
6. $\overrightarrow{PQ} = \langle 3-1, 4-2\rangle = \langle 2, 2\rangle$

$\overrightarrow{PR} = \langle -2-1, 2-2\rangle = \langle -3, 0\rangle$

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = 6k$$

$\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}| = \frac{1}{2}|6k| = \frac{6}{2} = 3$

7.



Area of PQSR

$= \Delta QSR + \Delta PQR$

$= \frac{1}{2}|\overrightarrow{SR}\times\overrightarrow{SQ}| + \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$\overrightarrow{SR} = \langle -5, 5, -1\rangle$ $\quad\overrightarrow{PQ} = \langle 1, -3, 2\rangle$

$\overrightarrow{SQ} = \langle -4, -2, 5\rangle$ $\quad\overrightarrow{PR} = \langle 0, 4, -4\rangle$

$$\overrightarrow{SR}\times\overrightarrow{SQ} = \begin{vmatrix} i & j & k \\ -5 & 5 & -1 \\ -4 & -2 & 5 \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ -2 & 5 \end{vmatrix} i - \begin{vmatrix} -5 & -1 \\ -4 & 5 \end{vmatrix} j + \begin{vmatrix} -5 & 5 \\ -4 & -2 \end{vmatrix} k$$

$$= \langle 23, 29, 30\rangle$$

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 0 & 4 & -4 \end{vmatrix} = \langle 4, 4, 4\rangle$$

$$\text{Area} = \frac{1}{2}\sqrt{23^2+29^2+30^2} + \frac{1}{2}\sqrt{4^2+4^2+4^2}$$

8. $\vec{r} = \langle 25, 0, 0\rangle$, $\quad\vec{F} = \langle 20\cos 40^\circ, 20\sin 40^\circ, 0\rangle$

$$\vec{r}\times\vec{F} = \begin{vmatrix} i & j & k \\ 25 & 0 & 0 \\ 20\cos 40^\circ & 20\sin 40^\circ & 0 \end{vmatrix} = 500\sin 40^\circ\ \vec{k}$$

$|\vec{r}\times\vec{F}| = 500\sin 40^\circ$, direction : out of the wall.