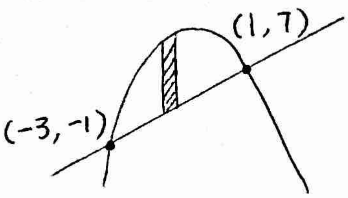
Example 1



By setting $-x^2+8=2x+5$, we get the intersection points

$$x=-3 \quad \text{or} \quad x=1$$

One stick: $(y_{Top}-y_{Bottom})\,dx$

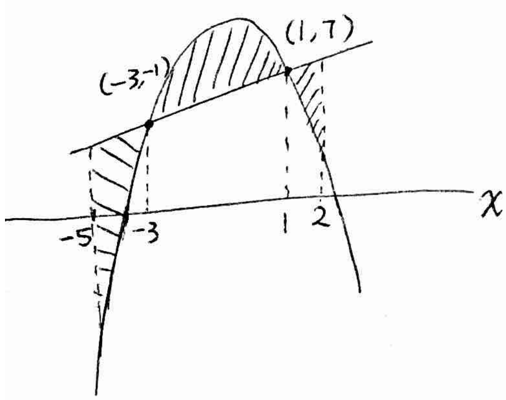
$$= (-x^2+8-(2x+5))\,dx$$

Thus, the area is

$$\int_{-3}^{1} -x^2+8-(2x+5)\,dx$$

$$= -\frac{1}{3}x^3-x^2+3x\Big|_{-3}^{1} = \frac{32}{3}$$

Example 2



On $[-5,-3]$, $y_{Top}=2x+5$

$y_{Bottom}=-x^2+8$

On $[-3,1]$, the area is given by Example 1

On $[1,2]$, $y_{Top}=2x+5$, $y_{Bottom}=-x^2+8$

Thus, the area is

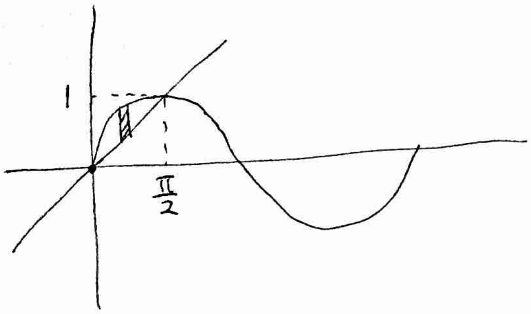
$$\int_{-5}^{-3}(2x+5-(-x^2+8))\,dx+\int_{-3}^{1}(-x^2+8-(2x+5))\,dx$$

$$+\int_{1}^{2}(2x+5-(-x^2+8))\,dx$$

$$=\frac{1}{3}x^3+x^2-3x\Big|_{-5}^{-3}+\left(-\frac{1}{3}x^3-x^2+3x\right)\Big|_{-3}^{1}+\left(\frac{1}{3}x^3+x^2-3x\right)\Big|_{1}^{2}$$

$$=\frac{71}{3}$$

Example 3:

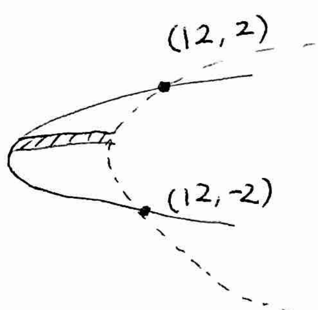


By setting $\sin x = \frac{2x}{\pi}$ where $x \geq 0$, we get $x = 0$, $x = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \left(\sin x - \frac{2x}{\pi}\right) dx$$

$$= -\cos x - \frac{x^2}{\pi} \Big|_0^{\frac{\pi}{2}} = 1 - \frac{\pi}{4}$$

Example 4



By setting $3y^2 = 8 + y^2$, we get the intersection points.

$y^2 = 4$ so $y = \pm 2$

Correspondingly,

When $y = 2$, $x = 12$

$y = -2$, $x = 12$

One "stick" area: $(x_{right} - x_{left})\, d\cancel{x}y$

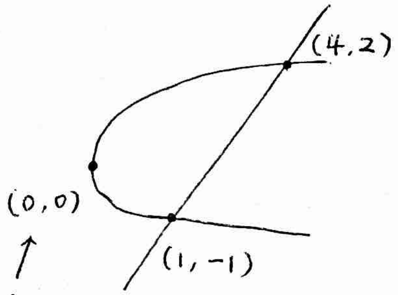
$$= 8 + y^2 - 3y^2\, dy$$

Thus

$$\text{Area} = \int_{-2}^{2} 8 + y^2 - 3y^2\, dy$$

$$= 8y - \frac{2}{3}y^3 \Big|_{-2}^{2} = \frac{64}{3}$$

Example 5:



Vertex of the parabola

By setting $\begin{cases} x = y^2 \\ y = x-2 \end{cases}$, we get the intersection points.

$$y^2 = y+2$$

So $y=-1$ or $y=2$

When $y=-1$, $x=1$

When $y=2$, $x=4$

Method 1: We use the horizontal sticks. Note that the right "wall" is the line, the left "wall" is the parabola.

$$\int_{-1}^{2} (x_{right} - x_{left})\, dy$$

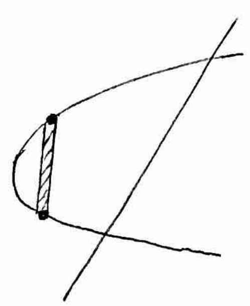
$x_{right} = y+2$ from the line

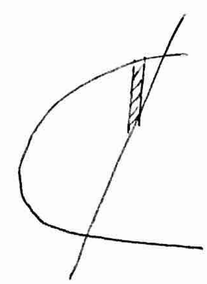
$x_{left} = y^2$

$$= \int_{-1}^{2} (y+2-y^2)\, dy$$

$$= \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^{2} = \frac{9}{2}$$

Method 2: We use the vertical sticks. Note that





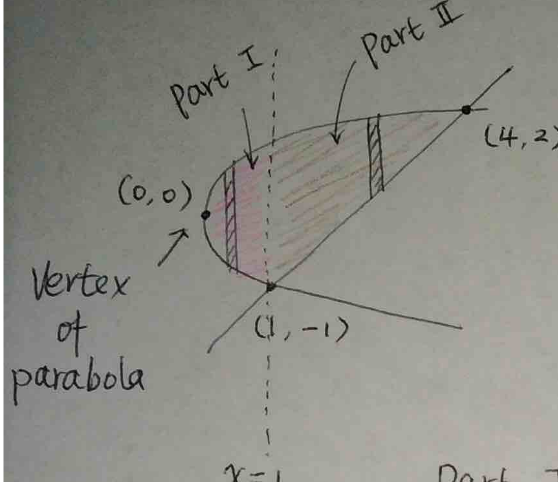
"Top": the top half of parabola

"Bottom": the lower half of parabola

"Top": the top half of parabola

"Bottom": the line

The "walls" change! We need to find the boundary line



Thus, we divide the area into two parts:

x runs from 0 to 1.

x runs from 1 to 4

Note that the "walls" don't change in each of the two parts. (pink shaded vs Brown shaded)

Part I: from $x = y^2$, we get $y = \pm\sqrt{x}$

$y_{Top} = \sqrt{x}$, $y_{Bottom} = -\sqrt{x}$

$$\text{Area of Part I} = \int_0^1 (y_{Top} - y_{Bottom})\,dx$$

$$= \int_0^1 2\sqrt{x}\,dx = \frac{4}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{4}{3}$$

Part II: $y_{Top} = \sqrt{x}$, $y_{Bottom} = x - 2$

$$\text{Area of Part II} = \int_1^4 (y_{Top} - y_{Bottom})\,dx$$

$$= \int_1^4 (\sqrt{x} - x + 2)\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 + 2x\Big|_1^4 = \frac{19}{6}$$

Thus, $\text{Area} = \frac{4}{3} + \frac{19}{6} = \frac{8+19}{6} = \frac{9}{2}$

They yield the same anwsers. However, we prefer the first method in this problem.