

Taylor Series of $f(x)$ at $x=a$.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

If $a=0$, it is Maclaurin Series of $f(x)$

Memorize the Common Maclaurin Series

$$f(x) = \frac{1}{1-x} : \sum_{n=0}^{\infty} x^n \quad R=1$$

$$f(x) = e^x : \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad R=\infty$$

$$f(x) = \sin x : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad R=\infty$$

$$f(x) = \cos x : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad R=\infty$$

$$f(x) = \arctan x : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \quad R=1$$

n^{th} -degree Taylor polynomial of $f(x)$ at $x=a$:

$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$$

The remainder $R_n(x) = f(x) - T_n(x) = \sum_{k=n+1}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k$

If $|f^{(n+1)}(x)| \leq M$, then $|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$

$$(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n \quad \text{for } |x| < 1, \quad \binom{a}{n} = \frac{a(a-1)\cdots(a-n+1)}{n!}, \quad \binom{a}{0} = 1$$