1. Differentiation and Integration of Power Series

If $f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$ has radius of convergence $R > 0$, then $f(x)$ is differentiable on the interval $(a-R, a+R)$.

(i) $f'(x) = \sum_{n=1}^{\infty} n C_n (x-a)^{n-1}$ (the radii of convergence is still R)

(ii) $\int f(x)\,dx = \text{Constant} + \sum_{n=0}^{\infty} \frac{C_n}{n+1} (x-a)^{n+1}$ (the radii of convergence is still R)

2. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for x values in $(-1, 1)$

↑

function represented as power series

$$\frac{1}{1+x} = \frac{1}{1-\underbrace{(-x)}_{☆}} = \sum_{n=0}^{\infty} ☆^n = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n, \quad -1<x<1$$

$$\frac{1}{1-\underbrace{x^4}_{☆}} = \sum_{n=0}^{\infty} (x^4)^n \qquad -1 < x^4 < 1 \quad \text{so} \quad x \in (-1, 1)$$

How about $\ln(1-x)$, $\frac{1}{(1-x)^2}$, ... ?

* Use differentiation or Integration to convert the function into $\frac{1}{1-☆}$

Then Anti-operate (Integration or differentiation) power series.

This is the strategy to represent function into power series.

Don't forget to label the radii and interval of convergence.