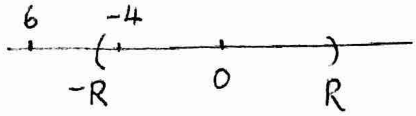
1. 

$\Sigma C_n x^n$ centered at 0

(A) $x = 1$, C

(B) $x = -3$, C

(C) $x = 8$, D

(D) $x = -9$, D

2. (A) $C_n = \dfrac{(-1)^n}{\sqrt{n^2+3}}$

$$\lim_{n\to\infty} \left|\frac{C_{n+1}}{C_n}\right| = \lim_{n\to\infty} \frac{\sqrt{n^2+3}}{\sqrt{(n+1)^2+3}} = \lim_{n\to\infty} \frac{\sqrt{n^2}}{\sqrt{n^2}} = 1 =: L$$

$\therefore R = \frac{1}{L} = 1$

Check endpoints : $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 1^n}{\sqrt{n^2+3}} \leftarrow x = 1$

Converge by Altenating Series Test

$x = -1$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt{n^2+3}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}} \sim \sum_{n=1}^{\infty} \frac{1}{n}$ Diverges.

Diverge by Limit Comparison Test and p-series Test

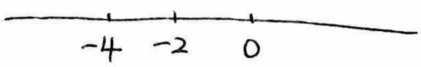
$\therefore$ Interval of Convergence : $(-1, 1]$. Absolute converges on $(-1, 1)$

(B) centered at -2.

$C_n = \dfrac{(-1)^{n+1}}{n 2^n}$

$$\lim_{n\to\infty} \left|\frac{C_{n+1}}{C_n}\right| = \lim_{n\to\infty} \frac{n 2^n}{(n+1) 2^{n+1}} = \frac{1}{2} \lim_{n\to\infty} \frac{n}{n+1} = \frac{1}{2}$$

so $R = 2$.



Check endpoints : $x = -4$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-2)^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{-1}{n} = -\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (p-series, p=1)

$x = 0$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(+2)^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges, by Alternating Series Test

$\therefore$ Interval of Convergence : $(-4, 0]$

Absolute convergence on $(-4, 0)$

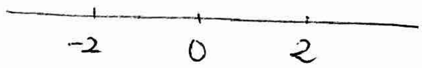
(c) centered at 0

$$C_n = \frac{n!}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$$

$$\lim_{n\to\infty} \frac{C_{n+1}}{C_n} = \lim_{n\to\infty} \frac{(n+1)!}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)} \cdot \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n!}$$

$$= \lim_{n\to\infty} \frac{n+1}{2n+1} = \frac{1}{2}$$

so $R = 2$.



Check Endpoints :

$x = 2$, $\displaystyle\sum_{n=1}^{\infty} \frac{n!\, 2^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$ diverges by Test for Divergence.

since $\displaystyle\lim_{n\to\infty} \frac{n!\, 2^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

$$\geq \lim_{n\to\infty} \frac{(2 \cdot 1) \cdot (2 \cdot 2) \cdot \cdots \cdot (2 \cdot n)}{(2 \cdot 1 - 1) \cdot (2 \cdot 2 - 1) \cdot \cdots \cdot (2 \cdot n - 1)} \geq 1.$$

$x = -2$, similar, diverges.

$\therefore$ Interval of Converges : $(-2, 2)$

Absolute Convergence on $(-2, 2)$

(D) Let $a_n = \dfrac{x^n}{n^4+2}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^4+2} \cdot \frac{n^4+2}{x^n} \right|$$

$$= |x| \lim_{n\to\infty} \frac{n^4+2}{(n+1)^4+2} = |x| < 1 \quad \text{(by ratio test, we need } |x|<1)$$

$\therefore \quad R = 1.$

$x=1$: $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4+2} = \sum_{n=1}^{\infty} \frac{1}{n^4+2}$ Converges by LCT, comparable to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$

$x=-1$: $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4+2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+2}$. Converges since $\displaystyle\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n^4+2}\right| = \sum_{n=1}^{\infty}\frac{1}{n^4+2}$ converges

$\therefore$ Interval of convergence : $[-1, 1]$

(E) $$\sum_{n=0}^{\infty} \frac{(2x+3)^n}{n!} = \sum_{n=0}^{\infty} \frac{(2(x+\frac{3}{2}))^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n (x+\frac{3}{2})^n}{n!}$$

$C_n = \dfrac{2^n}{n!}$, $\quad C_{n+1} = \dfrac{2^{n+1}}{(n+1)!}$

$$\lim_{n\to\infty} \left|\frac{C_{n+1}}{C_n}\right| = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0$$

So the radius $R = \infty$.

Interval of convergence : $(-\infty, \infty)$.

3. Check by yourself the radius of convergence is 5 for all the three series. (using the similar method as the previous problems).

Check the endpoints :

(A) $x=5$: $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{5^n} = \sum_{n=1}^{\infty} \frac{5^n}{5^n} = \sum_{n=1}^{\infty} 1$ diverges by TFD

$x=-5$: $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{5^n} = \sum_{n=1}^{\infty} \frac{(-5)^n}{5^n} = \sum_{n=1}^{\infty} (-1)^n$ diverges by TFD.

(B) $x=5$. $\sum_{n=1}^{\infty} \frac{x^n}{n5^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges, p-series $p=1$

$x=-5$, $\sum_{n=1}^{\infty} \frac{x^n}{n5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges, by AST

(c). $x=5$, $\sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, p-series $p=2$

$x=-5$, $\sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges, since $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.