Justify if a series is convergent or divergent:

1. Test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

   If $\lim_{n\to\infty} a_n = 0$, continue

2. Check the type of the series

   - Geometric Series: Σa_n converges if and only if $|r| < 1$
   - p series: Converges if and only if $p > 1$.
   - Telescoping series: Calculate partial sum and get the limit
   - Other series: other tests.

3. Other Convergence Tests

   - Alternating Series Test

     $$\sum_{n=1}^{\infty} (-1)^n b_n \quad \text{or} \quad \sum_{n=1}^{\infty} (-1)^{n+1} b_n \quad \text{where } b_n > 0$$

     If b_n decreases, that is $b_{n+1} \leq b_n$ for all n, and $\lim_{n\to\infty} b_n = 0$, then the series converges.

     * Alternating Series Error Estimation: $|R_k| = |s - s_k| \leq b_{k+1}$

   - Ratio Test

     $$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

     If $L < 1$, then Σa_n is absolutely convergent

     (i.e., $\Sigma |a_n|$ is convergent, implying Σa_n is convergent)

     If $L > 1$ or ∞, then Σa_n is divergent

     If $L = 1$, inconclusive.

If $\Sigma |a_n|$ converges, Σa_n is absolutely convergent.

$\Sigma |a_n|$ converges $\Rightarrow$ Σa_n converges

$(\not\Leftarrow)$

Example: $\Sigma \frac{(-1)^n}{n}$ is convergent by Alternating Series Test

but $\Sigma \left|\frac{(-1)^n}{n}\right| = \Sigma \frac{1}{n}$ is divergent by p-series test.

In this case, $\Sigma \frac{(-1)^n}{n}$ converges conditionally.