Summary for Sec 8.3

$\sum a_n$ converges $\Rightarrow$ $\lim_{n\to\infty} a_n = 0$

$(\not\Leftarrow)$

- Test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges

- Integral Test for Convergence

  $f(x)$ is continuous, positive, decreasing such that $f(n) = a_n$.

  $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

- p-series

  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$

- Error estimate with Integral Test

  $$\int_{n+1}^{\infty} f(x)\,dx \leq S - S_n \leq \int_n^{\infty} f(x)\,dx$$

- The Comparison Test.

  $0 \leq b_n \leq a_n$

  If $\sum a_n$ converges, then $\sum b_n$ converges

  If $\sum b_n$ diverges, then $\sum a_n$ diverges

- The Limit Comparison Test

  $\{a_n\}$, $\{b_n\}$ positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c \neq 0$, then

  <u>either</u> <u>both</u> $\sum a_n$ and $\sum b_n$ converge <u>or</u> <u>both</u> diverge.