

8.2 Series

• Partial Sum of the sequence $\{a_n\}_{n=1}^{\infty} : S_k = \sum_{n=1}^k a_n$

• $\lim_{k \rightarrow \infty} S_k$ exists $\Rightarrow \sum a_n$ the series converges

• Telescoping Series (Example $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$)

• Geometric Series $\sum ar^n = a \sum r^n$ a, r are constants

Converges when $|r| < 1$

$$\sum ar^n = \frac{\text{The first item}}{1-r}$$

• Properties of Series

$$\sum a_n, \sum b_n \text{ Converge} \Rightarrow \sum (a_n \pm b_n) = \sum a_n \pm \sum b_n$$

$$\sum (c a_n) = c \sum a_n$$

c is a constant

• Test for Divergence

$\lim a_n \neq 0$ or $\lim a_n$ DNE $\Rightarrow \sum a_n$ diverges

($\sum a_n$ converges $\Rightarrow \lim a_n = 0$)

Example of $\lim a_n = 0$ $\nRightarrow \sum a_n$ converges: Harmonic Series

$\sum \frac{1}{n}$, $\frac{1}{n} \rightarrow 0$, but $\sum \frac{1}{n}$ diverges