1. $a_n = S_n - S_{n-1} = (1-\frac{1}{n^2}) - (1-\frac{1}{(n-1)^2}) = \frac{1}{(n-1)^2} - \frac{1}{n^2}$

$\Sigma a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty} 1-\frac{1}{n^2} = 1$, so Σa_n converges and the value is 1

2. (a) No. "The item goes to 0" doesn't imply "the series converge"

(b) No

(c) Harmonic series

3.

The height of these triangles is the same, 0.5.

The base of these trianges: $\ldots, \frac{1}{8}-\frac{1}{16}, \frac{1}{4}-\frac{1}{8}, \frac{1}{2}-\frac{1}{4}, 1-\frac{1}{2}$

ie. $\frac{1}{2^0}-\frac{1}{2^1}, \frac{1}{2^1}-\frac{1}{2^2}, \frac{1}{2^2}-\frac{1}{2^3}, \frac{1}{2^3}-\frac{1}{2^4}, \ldots$

Total Area of these triangles:

$$\frac{1}{2}\cdot 0.5 \left[(\frac{1}{2^0}-\frac{1}{2^1}) + (\frac{1}{2^1}-\frac{1}{2^2}) + (\frac{1}{2^2}-\frac{1}{2^3}) + \cdots \right]$$

$$= \frac{1}{4}\sum_{n=0}^{\infty}(\frac{1}{2^n}-\frac{1}{2^{n+1}}) = \frac{1}{4}\left[\sum_{n=0}^{\infty}\frac{1}{2^n} - \sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\right] = \frac{1}{4},$$

where $\sum_{n=0}^{\infty}\frac{1}{2^n} = \frac{\frac{1}{2^0}}{1-\frac{1}{2}} = 2$ Geometric series with $r=\frac{1}{2}$

$\sum_{n=0}^{\infty}\frac{1}{2^{n+1}} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

4. (a) D, because $\lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}} = 1 \neq 0$, then by Test for Divergence

(b) D, because $\lim_{n\to\infty}\underbrace{\sqrt{4n^2+1}-n}_{\text{"}\infty-\infty\text{"}} = \lim_{n\to\infty}\frac{(\sqrt{4n^2+1})^2-n^2}{\sqrt{4n^2+1}+n} = \lim_{n\to\infty}\frac{3n^2+1}{\sqrt{4n^2+1}+n} = \infty \neq 0,$

then by Test for Divergence

(c) $\sum_{n=2}^{\infty}e^{3-2n} = \sum_{n=2}^{\infty}\frac{e^3}{e^{2n}} = e^3\sum_{n=2}^{\infty}\frac{1}{e^{2n}} = e^3\sum_{n=2}^{\infty}(\frac{1}{e^2})^n$ Geometric $r=\frac{1}{e^2}<1$

Converges, and $\sum_{n=2}^{\infty}e^{3-2n} = \frac{e^3\cdot(\frac{1}{e^2})^2}{1-\frac{1}{e^2}} = \frac{e}{e^2-1}$

(d) $\displaystyle\sum_{n=3}^{\infty} \frac{(-\pi)^n}{3^{2n-1}}$

$$= \sum_{n=3}^{\infty} \frac{(-\pi)^n \cdot 3}{3^{2n-1} \cdot 3} = 3 \sum_{n=3}^{\infty} \frac{(-\pi)^n}{3^{2n}} = 3 \sum_{n=3}^{\infty} \frac{(-\pi)^n}{9^n} = 3 \sum_{n=3}^{\infty} \left(-\frac{\pi}{9}\right)^n$$

Geometric, $r = -\frac{\pi}{9}$, $|r| = \frac{\pi}{9} < 1$

Converges, and the value is $3 \cdot \dfrac{\left(-\frac{\pi}{9}\right)^3}{1-\left(-\frac{\pi}{9}\right)} = 3 \cdot \dfrac{-\left(\frac{\pi}{9}\right)^3}{1+\frac{\pi}{9}}$

(e) Observe $\dfrac{-\frac{5}{3}}{\frac{25}{9}} = -\frac{3}{5}$, $\dfrac{1}{-\frac{5}{3}} = -\frac{3}{5}$, ...

Geometric Series, Converges.

The value is

$$\frac{\frac{25}{9}}{1-\left(-\frac{3}{5}\right)} = \frac{125}{72}$$