- Geometric Sequence $\{r^n\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} r^n = \begin{cases} 0, & \text{if } |r|<1 \\ 1, & \text{if } r=1 \\ \text{DNE}, & \text{if } |r|>1 \text{ or } r=-1 \end{cases}$$

- $\{a_n\}$ Conv. $\{b_n\}$ Conv.

  $\{a_n \pm b_n\}$, $\{a_n b_n\}$, $\{\frac{a_n}{b_n}\}$ with $\lim_{n\to\infty} b_n \neq 0$ all Conv.

- Find $\lim_{n\to\infty} a_n$

  Tools: L'Hopital Rule, Squeezing Theorem.

  - $\ln n \ll n^{\alpha}$ with $\alpha>0 \ll a^n$ with $a>1 \ll n! \ll n^n$

    (Meaning: $a_n \ll b_n$ means $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$) You can check it by L'Hopital rule or sequeezing theorem.

  - $\lim_{n\to\infty} (1+\frac{a}{n})^{bn} = e^{ab}$, a, b are numbers.

    (Apply L'Hopital Rule)

- Bounded + Monotone sequence Converges

- Recursively defined sequence

  Mathematical Induction:

  Statement: $P(n)$

  Check: (i) $P(0)$ or $P(1)$ for initial point

  (ii) Suppose $P(k)$ true. Check $P(k+1)$ true or not.

  Check Monotonicity (Increasing or Decreasing): Derivative

  Mathematical Induction.

- $\lim_{n\to\infty} |a_n| = 0 \Leftrightarrow \lim_{n\to\infty} a_n = 0$