Gateway Questions

How to Evaluate Functions at a Value Using the Rules

- Identify the independent variable in the rule of function.
- Replace the independent variable with big parenthesis.
- Plug in the input that needs to be evaluated inside the big parenthesis.

1. Evaluate the function $f(x) = 3x^2 - 3x + 1$ for $x = -1$.

Solution: $f(-1) = 3\left(-1\right)^2 - 3\left(-1\right) + 1 = \boxed{7}$

2. Evaluate the function $f(x) = 3x^2 - 3$ for $x = -1$.

Solution: $f(-1) = 3\left(-1\right)^2 - 3 = \boxed{0}$

3. Evaluate the function $f(x) = 3x^2 - 6x$ for $x = b - 1$.

Solution: $f(b-1) = 3\left(b-1\right)^2 - 6\left(b-1\right) = 3(b^2 - 2b + 1) - 6\left(b-1\right) = \boxed{3b^2 - 12b + 9}$

4. Evaluate the function $g(t) = 3t^2 - 3$ for $t = a + h$.

Solution: $g(a+h) = 3\left(a+h\right)^2 - 3 = 3(a^2 + 2ah + h^2) - 3 = \boxed{3a^2 + 6ah + 3h^2 - 3}$

5. Evaluate the function $v(t) = 3t + 3$ for $t = a + h$.

Solution: $f(a+h) = 3\left(a+h\right) + 3 = \boxed{3a + 3h + 3}$

6. Evaluate the function $g(t) = 6t - 3$ for $t = d - 2$.

Solution: $g(d-2) = 6\left(d-2\right) - 3 = \boxed{6d-15}$

7. Evaluate the function $f(x) = 6x + 3$ for $x = t + 2$.

Solution: $f(t+2) = 6\left(t+2\right) + 3 = \boxed{6t+15}$

8. Evaluate the function $f(y) = \dfrac{6y-1}{y}$ for $y = c + 3$.

Solution: $f(c+3) = \dfrac{6\left(c+3\right) - 1}{\left(c+3\right)} = \boxed{\dfrac{6c+17}{c+3}}$

9. Evaluate the function $f(y) = \dfrac{3y+3}{3y-3}$ for $y = m + k$.

Solution: $f(m+k) = \dfrac{3\left(m+k\right) + 3}{3\left(m+k\right) - 3} = \boxed{\dfrac{3m+3k+3}{3m+3k-3}}$

10. Evaluate the function $h(s) = 3 - s - \dfrac{1}{2}s^2$ for $s = j - 2$.

Solution: $h(c+3) = 3 - \left(j-2\right) - \dfrac{1}{2}\left(j-2\right)^2 = 3 - j + 2 - \dfrac{1}{2}\left(j^2 - 4j + 4\right) = (3+2-2) + (-j+2j) - \dfrac{1}{2}j^2 = \boxed{3 + j - \dfrac{1}{2}j^2}$

Solving Equations with Multiple Parameters: PreCalculus Version)

If the desired variable only appears to power of one, then follow the following process.

Isolate the Variable: First manipulate both sides so that each side clearly consists of different terms. For example, if one or both sides are quotient expressions, multiply both sides by each factor in denominator, Multiply all factors through and eliminate square roots. Add or subtract terms on both sides of the equation, make all terms on one sides contain the desirable variable and all terms on the other side do not contain that variable.

Factor the Variable: If the desirable variable still appears to power one only, you can factor the variable on one side.

Divide: Divide both sides by what multiplied the desirable variable.

11. Solve $P = S - Srt$ for r.

Solution:

Isolate the Variable: $Srt = S - P$

Factor the Variable: $r(St) = S - P$

Divide: $\boxed{r = \frac{S-P}{St}}$

12. Solve $2rx + 5 = 6(r - x)$ for x.

Solution:

Isolate the Variable:

$2rx + 5 = 6r - 6x \implies 2rx + 6x = 6r - 5$

Factor the Variable: $x(2r + 6) = 6r - 5$

Divide: $\boxed{x = \frac{6r-5}{2r+6}}$

13. Solve $\frac{1}{f} = \frac{3}{d_0} + \frac{5}{d_1}$ for f.

Solution:

Isolate the Variable: *Multiply by* fd_0d_1: $d_0d_1 = 3fd_1 + 5fd_0$.

Factor the Variable: $d_0d_1 = f(3d_1 + 5d_0)$

Divide: $f = \dfrac{d_0 d_1}{5d_0 + 3d_1}$

Also acceptable for Gateway Exam is: $\boxed{f = \dfrac{1}{\dfrac{3}{d_0} + \dfrac{5}{d_1}}}$

14. Solve $3ax - 5d = b(x - a)$ for x.

Solution:

Isolate the Variable:

$3ax - 5d = bx - ab \implies 3ax - bx = 5d - ab$

Factor the Variable: $x(3a - b) = (5d - ab)$

Divide: $\boxed{x = \dfrac{(5d - ab)}{(3a - b)}}$

15. Solve $v = \dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$ for v_1.

Solution:

Isolate the Variable:

$v(1 + \dfrac{v_1 v_2}{c^2}) = v_1 + v_2 \implies v + \dfrac{v v_1 v_2}{c^2} = v_1 + v_2$

$\implies \dfrac{v v_1 v_2}{c^2} - v_1 = v_2 - v$

Factor the Variable:

$v_1 \left(\dfrac{v v_2}{c^2} - 1 \right) = v_2 - v$

Divide: *Acceptable for Gateway Exam:* $\boxed{v_1 = \dfrac{v_2 - v}{\left(\dfrac{v v_2}{c^2} - 1 \right)}}$ *OR* $\boxed{v_1 = \dfrac{c^2(v_2 - v)}{v v_2 - c^2}}$

16. Solve $v = \dfrac{d + e}{1 + \frac{de}{c^2}}$ for e.

Solution:

Isolate the Variable:

$v(1 + \dfrac{de}{c^2}) = d + e \implies v + \dfrac{dev}{c^2} = d + e$

$$\frac{dev}{c^2} - e = d - v$$

Factor the Variable:

$$e\left(\frac{dv}{c^2} - 1\right) = d - v$$

Divide:

This is good enough for gateway: $\boxed{e = \dfrac{d - v}{\left(\dfrac{dv}{c^2} - 1\right)}}$ or $\boxed{e = \dfrac{c^2(d - v)}{dv - c^2}}$

17. Solve $x + y = \sqrt{x^2 + y^2 + 3}$ for y.

Solution:

Isolate the Variable:

Eliminate the radical: $(x + y)^2 = x^2 + y^2 + 3$

$$\underset{\text{Binomial Expansion}}{\Longrightarrow} \underbrace{x^2}_{\text{Subtract}} + \overbrace{y^2}^{\text{Subtract}} + 2xy = \underbrace{x^2}_{\text{Subtract}} + \overbrace{y^2}^{\text{Subtract}} + 3$$

$$\Longrightarrow\ 2xy = 3$$

Factor the variable:

$$y(2x) = 3$$

Divide: $\boxed{y = \dfrac{3}{2x}}$

18. Solve $Q_\omega = m_\omega c_\omega (T_f - T_\omega)$ for T_w.

Solution:

Isolate the Variable:

$$\underset{\text{Multiply through}}{\Longrightarrow} Q_\omega = m_\omega c_\omega T_f - m_\omega c_\omega T_\omega$$

$$\underset{\text{Add and subtract}}{\Longrightarrow} m_\omega c_\omega T_\omega = m_\omega c_\omega T_f - Q_\omega$$

Factor the Variable:

$$T_\omega (m_\omega c_\omega) = m_\omega c_\omega T_f - Q_\omega$$

Divide:

$$T_\omega = \frac{m_\omega c_\omega T_f - Q_\omega}{m_\omega c_\omega}$$

19. Solve $y - y_1 = m(x - x_1)$ for x.

Solution:

Isolate the Variable:

$y - y_1 = mx - mx_1 \implies mx = y - y_1 + mx_1$

Factor the Variable:

$x(m) = y - y_1 + mx_1$

Divide:

$$\boxed{x = \frac{y - y_1 + mx_1}{m}}$$

20. Solve $y - y_1 = m(x - x_1)$ for y.

Solution:

Isolate the Variable:

$y - y_1 = mx - mx_1 \implies y = y_1 + mx - mx_1$

Factor the Variable:

Done already: $y = y_1 + mx - mx_1$

Divide:

$$\boxed{y = y_1 + mx - mx_1}$$

21. Solve $\frac{x}{a} + \frac{y}{b} = 1$ for x.

Solution:

Isolate the Variable:

$\frac{x}{a} = 1 - \frac{y}{b}$

Factor the Variable:

$x(\frac{1}{a}) = 1 - \frac{y}{b}$

Divide:

$$\boxed{x = a - \frac{ay}{b}}$$

22. Solve $\dfrac{1}{x} + \dfrac{1}{y} = 1$ for y.

Solution:

Isolate the Variable:

$$\frac{1}{y} = 1 - \frac{1}{x}$$

Solve for the variable:

$$y = \frac{1}{1 - \frac{1}{x}}$$

Simplify:

$$y = \frac{1}{\frac{x-1}{x}}$$

$$\boxed{x = \frac{x}{x-1}}$$

Substitution Method for solving Equations. (PrecCaluclus version.)

Common Factors: Look for common factors to factor into simpler factors.

Relationship Between Exponents: Find if one of the exponents is twice or three times the other one. If there are two terms with variables and one exponent is twice the other one, expect a quadratic equation after substitution.

Substitution: The original variable to the smaller exponent becomes the New Variable.

Use one of the Types: At this point expect a quadratic or of the form A^2-B^2 or $A^3 \pm B^3$. Use quadratic formula $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ or the difference of squares formula $A^2 - B^2 = (A-B)(A+B)$ or the sum or difference of cubes formula $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$ to factor.

Factoring and/or Solving for the New Variable: Use each **factor** including any that may have been obtained in the first step and **SOLVE** for the New Variable.

Replace Back the Original Variable: For each value that you found, for the new variable, solve for the original variable. List all solutions with comma between them. In case no solution was possible, write NO SOLUTION. *On Gateway exam, give all exact solutions (square roots, fractions and so on.) Values such as* 2^5 *is accepted as well.*

Eliminate Extraneous Solutions: Plug back in the original equation and eliminate any extraneous solution that has been generated in the process.

23. Solve $x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 15 = 0$ for x.

Solution:

Common Factors: This one doesn't have an obvious common factor.

Relationship Between Exponents: $\frac{1}{3} = 2\left(\frac{1}{6}\right)$. So $x^{\frac{1}{3}} = \left(x^{\frac{1}{6}}\right)^2$.

Substitution: Let $y = x^{\frac{1}{6}}$. Replace $y^2 + 2y - 15 = 0$.

Use one of the Types: This can be factored easily but also the quadratic formula works.

Factoring and/or Solving for the New Variable: By quadratic formula, roots are:

$$y = \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)} = \begin{cases} y_1 = 3 \\ y_2 = -5 \end{cases}.$$

Replace Back the Original Variable: $x^{\frac{1}{6}} = 3$ ✓ and $x^{\frac{1}{6}} = -5$ ✗

$\Downarrow$ $\boxed{x = 3^6 = 729}$

$\Downarrow$ No solution for this one

24. Solve $(h-1)^{\frac{1}{3}} - 6(h-1)^{\frac{1}{6}} + 5 = 0$ for h.

Solution:

Common Factors: *This one doesn't have an obvious common factor.*

Relationship Between Exponents: $\frac{1}{3} = 2\left(\frac{1}{6}\right)$. *So* $(h-1)^{\frac{1}{3}} = \left((h-1)^{\frac{1}{6}}\right)^2$.

Substitution: $y^2 - 6y + 5 = 0$

Use one of the Types: *By quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)} = \begin{cases} y_1 = 5 \\ y_2 = 1 \end{cases}.$$

Replace Back the Original Variable:

$$y_1 = 5 \Downarrow (h-1)^{\frac{1}{6}} = 5 \Downarrow h - 1 = 15625 \Downarrow \boxed{h = 15626} ✓$$

$$y_2 = 1 \Downarrow (h-1)^{\frac{1}{6}} = 1 \Downarrow h - 1 = 1 \Downarrow \boxed{h = 2} ✓$$

Eliminate Extraneous Solutions:

None!

25. Solve $10x^{\frac{1}{2}} = x + 25$ for x.

Solution:

Common Factors: *None.*

Relationship Between Exponents: $1 = 2(\frac{1}{2}) \implies x = \left(x^{\frac{1}{2}}\right)^2$

Substitution: $y = x^{\frac{1}{2}} \implies y^2 = x$ *So the original equation becomes* $10y = y^2 + 25$.

Use one of the Types: *Quadratic formula for* $y^2 - 10y + 25 = 0$

Factoring and/or Solving for the New Variable:

$y^2 - 10y + 25 = 0 \implies y = 5$ *repeated root.*

Replace Back the Original Variable:

$y = x^{\frac{1}{2}} \implies x^{\frac{1}{2}} = 5 \implies \boxed{x = 25}$ ✓

Eliminate Extraneous Solutions: *None!*

26. Solve $14z^{\frac{3}{2}} + 29z^{\frac{5}{2}} - 15z^{\frac{7}{2}} = 0$ for z.

Solution:

Common Factors: *Factor* $z^{\frac{3}{2}}$ *to get:* $z^{\frac{3}{2}}(14+29z-15z^2) = 0 \implies z^{\frac{3}{2}} = 0 \implies \boxed{z_1 = 0}$ ✓

No substitution is needed.

Use one of the Types: *Quadratic formula since a quadratic equation is a factor.*

Factoring and/or Solving: *Use quadratic equation to find two more solutions: Order the terms from highest exponent to lowest exponent to find the coefficients:*

$\underbrace{-15}_{a} z^2 + \underbrace{29}_{b} z + \underbrace{14}_{c} = 0.$

Use the formula: $z = \dfrac{-29 \pm \sqrt{29^2 - 4(-15)(14)}}{2(-15)} = \begin{cases} \boxed{z_2 = \dfrac{7}{3}} ✓ \\ \boxed{z_3 = \dfrac{-2}{5}} ✓ \text{ for complex domain} \end{cases}$

Eliminate Extraneous Solutions: $z_3 = \dfrac{-2}{5}$ *is not in the domain for real numbers. (The real domain is all positive numbers because of the* $\sqrt{\ }$. z_3 *is a solution if we consider complex numbers though.)*

27. Solve $z^{\frac{7}{2}} - 8z^{\frac{5}{2}} = -16z^{\frac{3}{2}}$ for z.

Solution:

Common Factors: *Factor* $z^{\frac{3}{2}}$ *to get:* $z^{\frac{3}{2}}(z^{\frac{4}{2}} - 8z^{\frac{2}{2}} + 16z^{\frac{0}{2}}) = 0$

$\implies z^{\frac{3}{2}}(z^2 - 8z + 16) = 0 \implies z^{\frac{3}{2}} = 0 \implies \boxed{z_1 = 0}$ ✓

No substitution is needed.

Use one of the Types: *Quadratic formula since a quadratic equation is a factor.*

Factoring and/or Solving: *Use quadratic equation to find two more solutions: Order the terms from highest exponent to lowest exponent to find the coefficients:*

$\underbrace{\ \ }_{a} z^2 + \underbrace{-8}_{b} z + \underbrace{16}_{c} = 0.$

Use the formula: $z = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)} = \boxed{4}$ ✓

Eliminate Extraneous Solutions: *None!*

28. Solve $-21z^{\frac{1}{2}} + 41z^{\frac{3}{2}} - 10z^{\frac{5}{2}} = 0$ for z.

Solution:

Common Factors: *Factor* $z^{\frac{1}{2}}$ *to get:* $z^{\frac{1}{2}}(-21 + 41z - 10z^2) = 0 \implies z^{\frac{1}{2}} = 0 \implies \boxed{z_1 = 0}$ ✓

No substitution is needed.

Use one of the Types: *Quadratic formula since a quadratic equation is a factor.*

Factoring and/or Solving: *Use quadratic equation to find two more solutions: Order the terms from highest exponent to lowest exponent to find the coefficients:*

$\underbrace{-10}_{a} z^2 + \underbrace{41}_{b} z - \underbrace{21}_{c} = 0.$

Use the formula: $z = \dfrac{-41 \pm \sqrt{41^2 - 4(-10)(-21)}}{2(-10)} =$ ↗ $\boxed{z_2 = \dfrac{7}{2}}$ ✓ ↘ $\boxed{z_3 = \dfrac{3}{5}}$ ✓

Eliminate Extraneous Solutions: *There is none.*

29. Solve $(13x^2 - 11)^3 - (13x^2 - 11) = 0$ for x.

Solution:

Common Factors: $(13x^2 - 11)\Big((13x^2 - 11)^2 - 1\Big) = 0 \implies 13x^2 - 11 = 0 \implies x^2 = \dfrac{11}{13} \implies \boxed{x = \pm\sqrt{\dfrac{11}{13}}}$

Use one of the Types: *We are solving* $\Big((13x^2 - 11)^2 - 1\Big) = 0$ *so we can use factoring the difference of squares:* $A^2 - B^2 = (A - B)(A + B)$.

Factoring and/or Solving:

$\Big((13x^2 - 11) - 1\Big)\Big((13x^2 + 11) + 1\Big) = 0 \implies (13x^2 - 11 - 1) \quad (13x - 11 + 1) = 0$

$$\downarrow \qquad\qquad \Downarrow$$

$$x^2 = \frac{11 + 1}{13} = \frac{12}{13} \qquad x^2 = \frac{11 - 1}{13} = \frac{10}{13}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{x = \pm\sqrt{\frac{12}{13}}} \qquad \boxed{x = \pm\sqrt{\frac{10}{13}}}$$

30. Solve $(x+11)^3 = 125$ for x.

Solution:

Common Factors: *None!*

Relationship Between Exponents: *Only one term with variable to power 3.*

Substitution: $y = x + 11$ *to change the original equation to* $y^3 - 125 = 0$

Use one of the Types: *The difference of cubes.*

Factoring and/or Solving for the New Variable:

$$y^3 - 125 = (y-5)\left(y^2 + 5y + 25\right) = 0 \implies$$

$$y - 5 = 0 \qquad \left(y^2 + 5y + 25\right) = 0$$

$$\Downarrow \qquad \Downarrow$$

$$y - 5 = 0 \qquad y = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)}$$

$$\Downarrow \qquad y = \frac{-5 \pm (5)\sqrt{3}i}{2}$$

$$\Downarrow$$

$$x + 11 = y \implies \boxed{x = -6} \qquad x + 11 = y \implies \boxed{x = \frac{-27 \pm 5\sqrt{3}i}{2}}$$

Replace Back the Original Variable: ✓

Eliminate Extraneous Solutions: *None!*

31. Solve $(x-7)^3+27=0$ for x.

Solution:

Common Factors: *None!*

Relationship Between Exponents: *Only one term with variable to power 3.*

Substitution:

$y = x-7$ *to change the original equation to* $y^3+27=0$

Use one of the Types: *The sum of cubes.*

Factoring and/or Solving for the New Variable:

$$y^3+27=(y+3)\Big(y^2-3y+9\Big)=0 \implies$$

$$y+3=0 \qquad\qquad \Big(y^2-3y+9\Big)=0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$y+3=0 \qquad\qquad y=\frac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$$

$$\Downarrow \qquad\qquad y=\frac{3\pm(3)\sqrt{3}i}{2}$$

$$\Downarrow$$

$$x-7=y \implies \boxed{x=4} \qquad x-7=y \implies \boxed{x=\frac{17\pm 3\sqrt{3}i}{2}}$$

Replace Back the Original Variable: ✓

Eliminate Extraneous Solutions: *None!*

32. Solve $2x^{\frac{1}{2}} = 24$ for x.

Solution:

Common Factors: *None.*

Relationship Between Exponents: $1 = 2(\frac{1}{2})$

Substitution: $y = x^{\frac{1}{2}} \implies y^2 = x$ *So the original equation becomes* $2y = 24$.

Use one of the Types: *A linear equation.*

Factoring and/or Solving for the New Variable:

$$y = \frac{24}{2} \implies y = 12$$

Replace Back the Original Variable:

$$y = x^{\frac{1}{2}} \implies x^{\frac{1}{2}} = 12 \implies \boxed{x = 144} \checkmark$$

Eliminate Extraneous Solutions: *None!*

33. Solve $-10q + 41q^2 - 21q^3 = 0$ for q.

Solution:

- **Common Factors:** *The equation does not fit any of the familiar forms so we try the common factor.* $q(-10 + 41q - 21q^2) = 0 \implies \boxed{q_1 = 0}$
- *For the second factor, I recommend using quadratic formula since the equation is quadratic.*
- *Order the terms from highest exponent to lowest exponent to find the coefficients:*
  $$\underbrace{-21}_{a} q^2 + \underbrace{41}_{b} \; q \; \underbrace{-10}_{c} = 0.$$
- *Use the formula:* $q = \dfrac{-41 \pm \sqrt{41^2 - 4(-21)(-10)}}{2(-21)} =$ ↗ $\boxed{q_2 = \frac{5}{3}}$ ↘ $\boxed{q_3 = \frac{2}{7}}$

Radical Equations (PreCalculus version.)

Isolate one of the Radicals: Add or subtract terms from both sides of the equation to arrive at a equation with one radical on one side and the rest of the terms on the other side.

Both Sides to Power 2 (or whatever power that neutralizes the radical): Now that one radical is isolated, raise both side to power two. This way one of the radicals will be eliminated. Raising to power 2 for the other side of the equation MAY require a binomial calculation.

Eliminate the Next Radical if any: If the equation had more than one radical term, you may have to repeat the first and the second part.

Solve: When all radicals are eliminated, solve for the desired variable. A quadratic equation or other polynomial may be present at this stage.

Eliminate Extraneous Solutions: This stage of the work is really essential since, by squaring both side of the equation, extraneous solutions may have been produced which we need to eliminate. Plug in the solutions you found in the original equation.

34. Solve $\sqrt{x-2}-6=0$ for x

Solution:

Isolate one of the Radicals: $\sqrt{x-2}=6$.

Square Both sides: $x-2=36$

Eliminate the Next Radical if any: *Not this time.*

Solve: $x=38$

Eliminate Extraneous Solutions: *Plug in the original:* $\sqrt{38-2}-6=0$✓ *So* $\boxed{x=38}$ *is the solution.*

35. Solve $\sqrt{x-2}+6=7$ for x.

Solution:

Isolate one of the Radicals: $\sqrt{x-2}=1$.

Square Both sides: $x-2=1$

Eliminate the Next Radical if any: *Not this time.*

Solve: $x=3$

Eliminate Extraneous Solutions: *Plug in the original:* $\sqrt{3-2}+6=7$✓ *So* $\boxed{x=3}$ *is the solution.*

36. Solve $\sqrt{2-t}=6$ for t.

Solution:

Isolate one of the Radicals: *Already done!* $\sqrt{2-t}=6$

Square Both Sides: $2-t=36$

Eliminate the Next Radical if any: *Not this time.*

Solve: $t=-34$

Eliminate Extraneous Solutions: *Plug in the original:* $\sqrt{2-(-34)}=6$ ✓ *so* $\boxed{t=-34}$ *is the solution.*

37. Solve $c=4+\sqrt{4-c}$ for c.

Solution:

Isolate one of the Radicals: $\sqrt{4-c}=c-4$

Square Both Sides: $4-c=(c-4)^2$

$\implies 4-c=c^2-8c+16$

Eliminate the Next Radical if any: *Not this time.*

Solve: *Common form of quadratics:* $c^2\underbrace{-7}_{b}c+\underbrace{12}_{c}=0$

$$c=\frac{7\pm\sqrt{(-7)^2-4(1)(12)}}{2(1)}= \begin{cases} c_1=3 \\ c_2=4 \end{cases}$$

Eliminate Extraneous Solutions:

Plug in the original and check: for $c_1=3$, *we get* $3\neq 4+\sqrt{4-3}$ *The equality does NOT hold.* ✗

For $c_2=4$ *we get:* $4=4+\sqrt{4-4}$ ✓

The solution is $\boxed{c=4}$ ✓

This problem has an alternative method of solution:

$$\underbrace{\sqrt{4-c}+4-c=0}_{\text{Subtract } c \text{ from both sides}} \implies \underbrace{\sqrt{4-c}\left(\sqrt{4-c}+1\right)=0}_{\text{Factor the smaller exponent}} \implies \begin{cases} 4-c=0 \implies c=4 \\ 4-c=-1 \implies \text{No solution} \end{cases}$$

38. Solve $r = \sqrt{r-3} + 3$ for r.

Solution:

Isolate one of the Radicals: $\sqrt{r-3} = r - 3$

Square Both Sides: $r - 3 = (r-3)^2$

$\implies r - 3 = r^2 - 6r + 9$

Eliminate the Next Radical if any: *None this time.*

Solve:

Common form of quadratics: $r^2 \underbrace{-7}_{b} r + \underbrace{12}_{c} = 0$

$$c = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)} = \begin{matrix} \nearrow & c_1 = 3 \\ \searrow & c_1 = 4 \end{matrix}.$$

Eliminate Extraneous Solutions:

Plug in the original and check: for $c_1 = 4$, *we get* $4 = 3 + \sqrt{4-3}$ ✓

For $c_2 = 3$ *we get:* $3 = 3 + \sqrt{3-3}$ ✓

The solutions are $\boxed{r = 3 \text{ and } 4}$

This problem has an alternative method of solution:

$$\underbrace{\sqrt{r-3} + 3 - r = 0}_{\text{Subtract } c \text{ from both sides}} \implies \underbrace{\sqrt{r-3}\left(\sqrt{r-3} + 1\right) = 0}_{\text{Factor the smaller exponent}} \implies \begin{cases} r - 3 = 0 \implies r = 3 \\ r - 3 = 1 \implies r = 4 \end{cases}$$

39. Solve $4x = \sqrt{56x + 312}$ for x.

Solution:

Isolate one of the Radicals: *Radical is already isolated.*

Square Both Sides: $16x^2 = 56x + 312$

Eliminate the Next Radical if any: *No other Radical.*

Solve: *Quadratic form:* $16x^2 - 56x - 312 = 0$

Roots are $x = \dfrac{56 \pm \sqrt{(-56)^2 - (4)(16)(-312)}}{2(16)} = \begin{cases} 6.5 \\ -3 \end{cases}$

So the possible solutions: $x_1 = \dfrac{13}{2}$ *and* $x_2 = -3$.

Eliminate Extraneous Solutions:

$x_1 = \dfrac{13}{2}$, *plugging in the original results in* $2(\dfrac{13}{2}) = \sqrt{56(\dfrac{13}{2}) + 312}$. *This equality holds.* ✓

For $x_2 = -3$, *plugging in the original results in* $2(-3) = \sqrt{56(-3) + 312}$ *which does NOT hold because one side is negative and the other side is positive.* ✗

The solution is: $\boxed{\dfrac{13}{2}}$ ✓

40. Solve $b = \sqrt{12b - 35}$ for b.

Solution:

Isolate one of the Radicals: *Radical is already isolated.*

Square Both Sides: $b^2 = 12b - 35$

Eliminate the Next Radical if any: *No other Radical.*

Solve: *Quadratic form:* $b^2 - 12b + 35 = 0$

Roots are $b = \dfrac{12 \pm \sqrt{(-12)^2 - (4)(1)(35)}}{2(1)} = \begin{cases} 5 \\ 7 \end{cases}$

So the possible solutions: $b_1 = 5$ *and* $b_2 = 7$.

Eliminate Extraneous Solutions:

$b_1 = 5$, *plugging in the original results in* $5 = \sqrt{12(5) - 35}$. *This equality holds.* ✓

For $x_2 = 7$, *plugging in the original results in* $7 = \sqrt{12(7) - 35}$ *This equality holds.* ✓

The solutions are: $\boxed{5, 7}$ ✓

41. Solve $\sqrt{6-y}+\sqrt{5y+6}=6$ for y.

Solution:

Isolate one of the Radicals: $\sqrt{5y+6}=6-\sqrt{6-y}$

Square Both Sides:

$5y+6=\left(6-\sqrt{6-y}\right)^2 \underset{\text{Binomial Expansion}}{\implies} 5y+6=36-12\sqrt{6-y}+6-y$

$\underset{\text{Simplify}}{\implies} 12\sqrt{6-y}=36-6y \underset{\text{Simplify}}{\implies} 2\sqrt{6-y}=6-y$

Eliminate the Next Radical if any:

$\underset{\text{Square Both Sides Again}}{\implies} 4(6-y)=(6-y)^2 \underset{\text{Binomial Expansion}}{\implies} 4(6-y)=36-12y+y^2$

Solve:

The common form of quadratics: $y^2-8y+12=0$

$$y=\frac{8\pm\sqrt{8^2-4(1)(12)}}{2(1)} \quad \nearrow y_1=2 \quad \searrow y_2=6$$

Eliminate Extraneous Solutions: *Plug in and check.* $y=2$ *gives* $\sqrt{6-2}+\sqrt{5(2)+6}=6$ ✓ $y=6$ *gives* $\sqrt{6-6}+\sqrt{5(6)+6}=6$ ✓

- *The solutions are* $\boxed{y=2}$ *and* $\boxed{y=6}$

42. Solve $\sqrt{2x+7}-\sqrt{2x-9}=2$ for x.

Solution:

Isolate one of the Radicals: $\sqrt{2x+7}=\sqrt{2x-9}+2$

Square Both Sides:

$2x+7=\left(\sqrt{2x-9}+2\right)^2 \underset{\text{Binomial Expansion}}{\implies} 2x+7=2x-9+4\sqrt{2x-9}+4$

$\underset{\text{Simplify}}{\implies} \sqrt{2x-9}=3$

Eliminate the Next Radical if any: $\underset{\text{Square Both Sides Again}}{\implies} 2x-9=3^2$

Solve: $x=9$

Eliminate Extraneous Solutions: $\sqrt{2(9)+7}-\sqrt{2(9)-9}=2$ ✓

The solution is: $\boxed{x=9}$

43. Solve $\sqrt{m+7}+\sqrt{m-5}=6$ for m.

Solution:

Isolate one of the Radicals: $\sqrt{m+7}=6-\sqrt{m-5}$

Square Both Sides: $m+7=(6-\sqrt{m-5})^2 \underset{\text{Binomial Expansion}}{\Longrightarrow} m+7=36-12\sqrt{m-5}+m-5$

Eliminate the Next Radical if any:

$\sqrt{m-5}=2 \underset{\text{Square Both Sides Again}}{\Longrightarrow} m-5=4$

Solve: $m=9$

Eliminate Extraneous Solutions:

Plug in and check: $\sqrt{9+7}+\sqrt{9-5}=6$✓

The solution is $\boxed{m=9}$

Substitution Method for solving Equations. (Second set.)

44. Solve $a^4 - 7a^2 = -10$ for a.

Solution:

Common Factors: No common factors.

Relationship Between Exponents: $4 = 2(2)$ so $a^4 = (a^2)^2$

Substitution: Let $y = a^2 \implies y^2 = a^4$. Replace $y^2 - 7y = -10$

Use one of the Types: This one has an easy factoring method but I use the quadratic formula.

Factoring and/or Solving for the New Variable:

$$y = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)} = \begin{cases} y_1 = 2 \\ y_2 = 5 \end{cases}$$

Replace Back the Original Variable:

$$a^2 = y \implies \begin{cases} a^2 = 2 \implies \boxed{a = \pm\sqrt{2}} \checkmark \\ a^2 = 5 \implies \boxed{a = \pm\sqrt{5}} \checkmark \end{cases}$$

Eliminate Extraneous solutions: All answers work in the original equation.

45. Solve $9x^4 = 30x^2 - 25$ for x

Solution:

Common Factors: *None.*

Relationship Between Exponents: $4 = 2(2) \implies (x^2)^2 = x^4$.

Substitution: *Let* $y = x^2 \implies y^2 = x^4$. *So the new equation is* $9y^2 - 30y + 25 = 0$.

Use one of the Types: *Quadratic formula.* $9y^2 - 30y + 25 = 0$.

Factoring and/or Solving for the New Variable:

$$y = \frac{30 \pm \sqrt{(-30)^2 - 4(9)(25)}}{2(9)} = \frac{5}{3}$$

$\Downarrow$

Repeated Root

Replace Back the Original Variable:

$$y = x^2 \implies x^2 = \frac{5}{3} \implies \boxed{x = \pm\sqrt{\frac{5}{3}}} \checkmark$$

Eliminate Extraneous Solutions: *None!*

46. Solve $3x^4 - 13x^2 - 10 = 0$ for x.

Solution:

Common Factors: *No common factors.*

Relationship Between Exponents: $4 = 2(2) \implies (x^2)^2 = x^4$.

Substitution: *Let* $y = x^2 \implies y^2 = x^4$. *So the new equation is* $3y^2 - 13y - 10 = 0$.

Use one of the Types: *Quadratic equation.*

Factoring and/or Solving for the New Variable:

$$y = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(-10)}}{2(3)} = \begin{cases} y_1 = 5 \\ y_2 = -\dfrac{2}{3} \end{cases}$$

Replace Back the Original Variable: $x^2 = y$

$$x^2 = 5 \implies \boxed{x = \pm\sqrt{5}} \checkmark$$

$$x^2 = -\frac{2}{3} \implies \boxed{x = \pm\sqrt{\frac{2}{3}}i} \checkmark$$

Eliminate Extraneous Solutions: $\boxed{x = \pm\sqrt{5}}$ *and* $\boxed{x = \pm\sqrt{\frac{2}{3}}i}$ ✓ *are the solutions.*

47. Solve $2x^4 - 13x^2 + 15 = 0$ for x.

Solution:

Common Factors: *No common factors.*

Relationship Between Exponents: $4 = 2(2) \implies (x^2)^2 = x^4$.

Substitution: *Let* $y = x^2 \implies y^2 = x^4$. *So the new equation is* $2y^2 - 13y + 15 = 0$.

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{13 \pm \sqrt{(-13)^2 - 4(2)(15)}}{2(2)} = \begin{cases} y_1 = 5 \\ y_2 = \dfrac{3}{2} \end{cases}$$

Replace Back the Original Variable: $x^2 = y$

$$x^2 = 5 \implies \boxed{x = \pm\sqrt{5}} \checkmark$$

$$x^2 = \frac{3}{2} \implies \boxed{x = \pm\sqrt{\frac{3}{2}}} \checkmark$$

Eliminate Extraneous Solutions: *All four solutions are correct.*

48. Solve $x^4 - 8x^2 + 15 = 0$ for x.

Solution:

Common Factors: *None*

Relationship Between Exponents: $4 = 2(2) \implies x^4 = (x^2)^2$

Substitution: *Let* $y = x^2 \implies y^2 = x^4$ *The new equation is* $y^2 - 8y + 15 = 0$.

Use one of the Types: *Quadratic.*

Factoring and/or Solving for the New Variable:

$$y = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(15)}}{2(1)} = \begin{cases} y_1 = 3 \\ y_2 = 5 \end{cases}$$

Replace Back the Original Variable:

$$x^2 = y \begin{cases} x^2 = 3 \implies \boxed{x = \pm\sqrt{3}} \checkmark \\ x^2 = 5 \implies \boxed{x = \pm\sqrt{5}} \checkmark \end{cases}$$

Eliminate Extraneous Solutions: *None.*

49. Solve $(\frac{g-2}{g})^2 - 7(\frac{g-2}{g}) + 6 = 0$ for g.

Solution:

Common Factors: *None!*

Relationship Between Exponents: $2 = 2(1)$. *Variable in one term is the square of the variable in the other term.*

Substitution:

$y = \frac{g-2}{g}$ *By substituting in the original equation, we get* $y^2 - 7y + 6 = 0$

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)} = \begin{cases} \nearrow & y_1 = 1 \\ \searrow & y_2 = 6 \end{cases}$$

Replace Back the Original Variable:

$$y = \frac{g-2}{g}$$

$$\swarrow \quad \searrow$$

$\frac{g-2}{g} = 1 \Downarrow g - 2 = g \Downarrow 0 = 1$ ✗ $\Downarrow$ *No solution for this one*

$\frac{g-2}{g} = 6 \Downarrow g - 2 = 6g \Downarrow -2 = 5g \Downarrow \boxed{g = -\frac{2}{5}}$ ✓

Eliminate Extraneous Solutions: *All are correct.*

50. Solve $(\frac{f+3}{f})^2 - 10(\frac{f+3}{f}) + 9 = 0$ for f.

Solution:

Common Factors: *None!*

Relationship Between Exponents: $2 = 2(1)$. *Variable in one term is the square of the variable in the other term.*

Substitution: $y = \frac{f+3}{f}$ *By substituting in the original equation, we get* $y^2 - 10y + 9 = 0$

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(9)}}{2(1)} = \begin{cases} y_1 = 1 \\ y_2 = 9 \end{cases}$$

Replace Back the Original Variable:

$$y = \frac{f+3}{f}$$

$$\frac{f+3}{f} = 1 \Downarrow f+3 = f \Downarrow 0 = 1 ✗ \Downarrow$$

No solution for this one.

$$\frac{f+3}{f} = 9 \Downarrow f+3 = 9f \Downarrow 3 = 8f \Downarrow \boxed{f = \frac{3}{8}} ✓$$

Eliminate Extraneous Solutions: *All are correct!*

51. Solve $10(\frac{x+5}{x})^2 + 7(\frac{x+5}{x}) + 1 = 0$ for x.

Solution:

Common Factors: *None!*

Relationship Between Exponents: $2 = 2(1)$. *Variable in one term is the square of the variable in the other term.*

Substitution: $y = \frac{x+5}{x}$ *By substituting in the original equation, we get* $10y^2 + 7y + 1 = 0$

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(10)} = \begin{cases} \nearrow & y_1 = -\frac{1}{5} \\ \searrow & y_2 = -\frac{1}{2} \end{cases}$$

Replace Back the Original Variable:

$$y = \frac{x+5}{x}$$

$\swarrow$ $\searrow$

$$\frac{x+5}{x} = -\frac{1}{2} \Downarrow x+5 = -\frac{1}{2}x \Downarrow 2(x+5) = -x \Downarrow \boxed{x = -\frac{10}{3}} \checkmark$$

$$\frac{x+5}{x} = -\frac{1}{5} \Downarrow x+5 = -\frac{1}{5}x \Downarrow 5(x+5) = -x \Downarrow \boxed{x = -\frac{25}{6}} \checkmark$$

Eliminate Extraneous Solutions: *All are correct!*

52. Solve $6(\frac{g}{g+2})^2 - 7(\frac{g}{g+2}) + 1 = 0$ for g.

Solution:

Common Factors: *None!*

Relationship Between Exponents: $2 = 2(1)$. *Variable in one term is the square of the variable in the other term.*

Substitution: $y = \frac{g}{g+2}$ *By substituting in the original equation, we get* $6y^2 - 7y + 1 = 0$

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$$y = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(6)} = \begin{cases} y_1 = 1 \\ y_2 = \frac{1}{6} \end{cases}$$

Replace Back the Original Variable:

$$y = \frac{g}{g+2}$$

$$\frac{g}{g+2} = 1 \Downarrow g = g + 2 \Downarrow 0 = 1 ✗ \Downarrow$$

No solution for this one

$$\frac{g}{g+2} = \frac{1}{6} \Downarrow 6g = (g+2) \Downarrow 2 = 5g \Downarrow \boxed{g = \frac{2}{5}} ✓$$

Eliminate Extraneous Solutions: *All are correct.*

53. Solve $4(\frac{g}{g+4})^2 - 4(\frac{g}{g+4}) + 1 = 0$ for g.

Solution:

Common Factors: *None!*

Relationship Between Exponents: $2 = 2(1)$. *Variable in one term is the square of the variable in the other term.*

Substitution: $y = \frac{f+4}{f}$ *By substituting in the original equation, we get* $4y^2 - 4y + 1 = 0$

Use one of the Types: *Quadratic formula.*

Factoring and/or Solving for the New Variable:

$y = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(4)} = \quad \frac{1}{2}$ *Repeated.*

Replace Back the Original Variable:

$\frac{g}{g+4} = \frac{1}{2} \implies 2g = g + 4 \implies 1g = 4 \implies \boxed{g = \frac{4}{1}}$

Eliminate Extraneous Solutions: *All are correct!*

How to Solve Most Exponential Equations in PreCalculus

Using the Exponential Rules to simplify: If needed, use any of the rules (1) $e^x e^y = e^{x+y}$, (2) $\dfrac{e^x}{e^y} = e^{x-y}$, (3) $(e^x)^y = e^{xy}$, to create single exponential term on each side.

Setting an Equation Using the Exponents of Both Sides: Take logarithm of both side to get an equation without any exponential terms. In this step, you will use the rule $\ln(e^x) = x$.

Solve for the Variable: Solve the equation from previous step.

Extraneous Solutions: Eliminate all solutions that were generate as a result of solving the equation but are not a solution.

How to Solve Most Logarithmic Equations in PreCalculus

Using the Logarithmic Rules to Simplify: If needed, use any of the rules (1) $log(xy) = \log(x) + \log(y)$, (2) $\log\left(\dfrac{x}{y}\right) = \log(x) - \log(y)$, (3) $k\log(x) = \log\left(x^k\right)$, to create single logarithmic term on each side.

Setting an Equation Using the Exponents of Both Sides: Raise the base to power both side to get an equation without any logarithmic terms. In this step, you will use the rule $e^{\ln(x)} = x$.

Solve for the Variable: Solve the equation from previous step.

Extraneous Solutions: Eliminate all solutions that were generate as a result of solving the equation but are not a solution.

54. Solve $e^{4x}e^{4x} = (e^x)^x e^{-9}$ for x.

Solution:

Using the Exponential Rules to simplify:

$$\left.\begin{array}{ccc} e^{4x}e^{4x} & = & \underbrace{(e^x)^x}_{e^{x^2}} e^{-9} \\ \Uparrow & & \Uparrow \\ e^{8x} & = & e^{x^2-9} \end{array}\right\} \Longrightarrow e^{8x} = e^{x^2-9}.$$

Setting an Equation Using the Exponents of Both Sides: $\underset{\text{Take ln of both sides}}{\Longrightarrow} 8x = x^2 - 9$

Solve for the Variable: $\Longrightarrow x^2 - 8x - 9 = 0$

$$\underset{\text{Use quadratic formula}}{\Longrightarrow} x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-9)}}{2(1)} = \begin{array}{l} \nearrow \boxed{x_1 = -1} \\ \searrow \boxed{x_2 = 9} \end{array}$$

55. Solve $2^{7t+1} = 2^{t^2+11}$ for t.

Solution:

Using the Exponential Rules to simplify: *Not needed.*

Setting an Equation Using the Exponents of Both Sides: $\underset{\text{Take } \log_2 \text{ of both sides}}{\Longrightarrow} 7t + 1 = t^2 + 11$

Solve for the Variable: $\Longrightarrow t^2 - 7t + 10 = 0$

$$\underset{\text{Use quadratic formula}}{\Longrightarrow} t = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)} = \begin{cases} \nearrow \boxed{t_1 = 2} \\ \searrow \boxed{t_2 = 5} \end{cases}$$

56. Solve $7^{4r+1} = 7^{r^2}7^5$ for r.

Solution:

Using the Exponential Rules to simplify:

$$\left.\begin{array}{ccc} 7^{4r+1} & = & 7^{r^2}7^5 \\ & & \Uparrow \\ & & 7^{r^2+5} \end{array}\right\} \Longrightarrow 7^{4r+1} = 7^{r^2+5}$$

Setting an Equation Using the Exponents of Both Sides: $\underset{\text{Take } \log_7 \text{ of both sides}}{\Longrightarrow} 4r + 1 = r^2 + 5$

Solve for the Variable: $\Longrightarrow r^2 - 4r + 4 = 0$

$$\underset{\text{Use quadratic formula}}{\Longrightarrow} r = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)} = \boxed{2} \text{ repeated.}$$

57. Solve $(e^{2m})^{4m} = e^{3-2m}$ for m.

Solution:

Using the Exponential Rules to simplify:

$$\left.\begin{array}{ccc} (e^{2m})^{4m} & = & e^{3-2m} \\ \Uparrow & & \\ e^{8m^2} & & \end{array}\right\} \Longrightarrow e^{8m^2} = e^{3-2m}.$$

Setting an Equation Using the Exponents of Both Sides: $\underset{\text{Take ln of both sides}}{\Longrightarrow} 8m^2 = 3 - 2m$

Solve for the Variable: $\Longrightarrow 8m^2 + 2m - 3 = 0$

$$\underset{\text{Use quadratic formula}}{\Longrightarrow} m = \frac{-2 \pm \sqrt{2^2 - 4(8)(-3)}}{2(8)} = \begin{cases} \boxed{m_1 = -\frac{3}{4}} \\ \boxed{m_2 = \frac{1}{2}} \end{cases}$$

58. Solve $(7^{7x})^x = (7^{49})^x$ for x.

Solution:

Using the Exponential Rules to simplify:

$$\left.\begin{array}{ccc} (7^{7x})^x & = & (7^{49})^x \\ \Uparrow & & \Uparrow \\ 7^{7x^2} & = & 7^{49x} \end{array}\right\} \Longrightarrow 7^{7x^2} = 7^{49x}.$$

Setting an Equation Using the Exponents of Both Sides: $\underset{\text{Take } \log_7 \text{ of both sides}}{\Longrightarrow} 7x^2 = 49x$

Solve for the Variable: $\Longrightarrow 7x^2 - 49x = 0$

$$\underset{\text{Subtract } 49x \text{ and Factor } 7x}{\Longrightarrow} 7x(x-7) = 0 \begin{cases} \boxed{x_1 = 0} \\ \boxed{x_2 = 7} \end{cases}$$

59. Solve $\ln(7x-3) = \ln(23) + \ln(2)$ for x.

Solution:

Using the Logarithmic Rules to Simplify: $\ln(7x-3) = \ln(46)$

Setting an Equation Using the Exponents of Both Sides: $\implies e^{\ln(7x-3)} = e^{\ln(46)} \implies 7x-3 = 46$

Solve for the Variable: $\implies 7x = 49 \implies \boxed{x = 7}$ ✓

60. Solve $\ln(x+3) - \ln(x) = 1$ for x.

Solution:

Using the Logarithmic Rules to Simplify: $\ln\left(\dfrac{x+3}{x}\right) = 1$

Setting an Equation Using the Exponents of Both Sides: $\implies e^{\ln\left(\frac{x+3}{x}\right)} = e^1$

$\implies \left(\dfrac{x+3}{x}\right) = e$

Solve for the Variable: $\implies x + 3 = ex \implies (e-1)x = 3 \implies \boxed{x = \dfrac{3}{e-1}}$ ✓

61. Solve $\ln(x) = \ln(64) - 2\ln(x)$ for x.

Solution:

Using the Logarithmic Rules to Simplify:

$$\left.\begin{array}{rcl} \ln(x) & = & \ln(64) - \underbrace{2\ln(x)}_{\ln(x^2)} \\ & & \Uparrow \\ & & \ln\left(\dfrac{64}{x^2}\right) \end{array}\right\} \implies \ln(x) = \ln\left(\dfrac{64}{x^2}\right)$$

Setting an Equation Using the Exponents of Both Sides: $\implies e^{\ln(x)} = e^{\ln\left(\frac{64}{x^2}\right)}$

$\implies x = \left(\dfrac{64}{x^2}\right)$

Solve for the Variable: $\implies x^3 = 64 \implies \boxed{x = 4}$ ✓

62. Solve $\ln(4p) + \ln\left(p + \frac{7}{4}\right) = \ln(2)$ for p.

Solution:

Using the Logarithmic Rules to Simplify:

$$\underbrace{\underbrace{\ln(4p) + \ln\left(p+\frac{7}{4}\right)}_{\ln\left(4p\left(p+\frac{7}{4}\right)\right) \Uparrow \ln(4p^2+7p)} = \ln(2)}_{\ln(4p^2+7p)=\ln(2)}$$

Setting an Equation Using the Exponents of Both Sides: $\Longrightarrow e^{\ln\left(4p^2+7p\right)} = e^{\ln(2)}$

$\Longrightarrow 4p^2 + 7p = 2$

Solve for the Variable: $\Longrightarrow 4p^2 + 7p - 2 = 0$

$$\Longrightarrow p = \frac{-7 \pm \sqrt{7^2 - (4)(4)(-2)}}{2(4)}$$

$p_1 = -2$ ✗ $\Longleftarrow$ Not in the domain

$\boxed{p_2 = \frac{1}{4}}$ ✓

63. Solve $\ln(3x) + \ln\left(x - \frac{2}{3}\right) = \frac{1}{2}\ln(64)$ for x.

Solution:

Using the Logarithmic Rules to Simplify:

$$\underbrace{\underbrace{\ln(3x) + \ln\left(x-\frac{2}{3}\right)}_{\ln\left(3x\left(x-\frac{2}{3}\right)\right) \Uparrow \ln(3x^2-2x)} = \underbrace{\frac{1}{2}\ln(64)}_{\ln\left(64^{\frac{1}{2}}\right) \Uparrow \ln(8)}}_{\ln(3x^2-2x)=\ln(8)}$$

Setting an Equation Using the Exponents of Both Sides: $\Longrightarrow e^{\ln\left(3x^2-2x\right)} = e^{\ln(8)}$

$\Longrightarrow 3x^2 - 2x = 8$

Solve for the Variable: $\Longrightarrow 3x^2 - 2x - 8 = 0$

$$\Longrightarrow x = \frac{2 \pm \sqrt{(-2)^2 - (4)(3)(-8)}}{2(3)}$$

$x_1 = -\frac{4}{3}$ ✗ $\Longleftarrow$ Not in the domain

$\boxed{x_2 = 2}$ ✓

Function Operations

- Identify the outer and inner function. For example in $f \circ g$, f is the outer and g is the inner function.
- Write the outer and write big parenthesis whenever you see the independent variable.
- Write the inner function in every parenthesis.

64. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $(f - g)(4)$.

Solution: $(f-g)(4) = \left(4\right)^2 + 3 - \left(\sqrt{\left(4\right)} - 3\right) = 16 + 3 - 2 + 3 = \boxed{20}$

65. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $(f + g)(4)$.

Solution: $(f+g)(4) = \left(4\right)^2 + 3 + \left(\sqrt{\left(4\right)} - 3\right) = 16 + 3 + 2 - 3 = \boxed{18}$

66. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $(g - f)(4)$.

Solution: $(g-f)(4) = \sqrt{\left(4\right)} - 3 - \left(\left(4\right)^2 + 3\right) = 2 - 3 - 16 - 3 = \boxed{-20}$

67. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $(\frac{g}{f})(a)$.

Solution: $(\frac{g}{f})(a) = \dfrac{\left(\sqrt{\left(a\right)} - 3\right)}{\left(a\right)^2 + 3} = \boxed{\dfrac{\sqrt{a} - 3}{a^2 + 3}}$

68. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $(gf)(x)$.

Solution: $(gf)(x) = (\sqrt{x} - 3)(x^2 + 3) = \boxed{(\sqrt{x} - 3)(x^2 + 3)}$

69. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $3g(c)$.

Solution: $3g(c) = 3\left(\sqrt{\left(c\right)} - 3\right)) = \boxed{3\sqrt{c} - 9}$

70. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $2f(1)$.

Solution: $2f(1) = 2\left(\left(1\right)^2 + 3\right) = \boxed{8}$

71. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $g(f(x))$.

Solution: $g(f(x)) = \sqrt{x^2 + 3} - 3 = \boxed{\sqrt{x^2 + 3} - 3}$

72. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $g(f(x + y))$.

Solution: $g(f(x + y)) = \sqrt{\left(x + y\right)^2 + 3} - 3 = \boxed{\sqrt{x^2 + y^2 + 2xy + 3} - 3}$

73. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $g(f(\sqrt{2}))$.

Solution: $g(f(\sqrt{2})) = \sqrt{\left(\sqrt{2}\right)^2 + 3} - 3 = \boxed{\sqrt{5} - 3}$

74. Given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x} - 3$, find the value of $f(g(a+h))$.

Solution: $f(g(a+h)) = \boxed{\left(\sqrt{a+h} - 3\right)^2 + 3}$

75. Given $g(x) = \dfrac{1}{x+3}$ and $f(x) = \sqrt{x}$, find $f(g(x))$.

Solution: $f(g(x)) = \sqrt{\dfrac{1}{x+3}}$

76. Given $g(x) = \dfrac{x-1}{x+1}$ and $f(x) = x^2$, find $f(g(x))$.

Solution: $\boxed{f(g(x)) = \left(\dfrac{x-1}{x+1}\right)^2}$

77. Given $g(x) = 9x + \dfrac{2}{x-1}$ and $f(x) = 1 + 2x$, find $f(g(x))$.

Solution: $f(g(x)) = 1 + 2\left(9x + \dfrac{2}{x-1}\right) = \boxed{1 + 18x + \dfrac{4}{x-1}}$

78. Given $g(x) = \dfrac{3}{x} - x$ and $f(x) = \dfrac{x}{3} + x$, find $f(g(x))$.

Solution: *Note*: $f(x) \underset{\text{Factor } x}{=} x(\frac{1}{3} + 1) = \frac{4}{3}x$

$f(g(x)) = \dfrac{4}{3}\left(\dfrac{3}{x} - x\right) = \boxed{\dfrac{4}{x} - \dfrac{4x}{3}}$

79. Given $f(x) = \dfrac{3}{x} - x$ and $g(x) = \dfrac{x}{3} + x$, find $f(g(x))$.

Solution: *Note:* $g(x) \underset{\text{Factor } x}{=} x(\frac{1}{3} + 1) = \frac{4}{3}x$

$$f(g(x)) = \frac{3}{\left(\frac{4x}{3}\right)} - \left(\frac{4x}{3}\right) = \boxed{\frac{9}{4x} - \frac{4x}{3}}$$

80. Given $f(x) = x^2 + 4x - 5$ and $g(x) = x - c$, find $f(g(x))$.

Solution: $f(g(x)) = \left(x - c\right)^2 + 4\left(x - c\right) - 5 = \left(x^2 - 2cx + c^2\right) + \left(4x - 4c\right) - 5 =$
$\boxed{x^2 + (4 - 2c)x + (c^2 - 4c - 5)}$

81. Given $g(x) = 5x^2 - 2$ and $f(x) = \sqrt{x} + 1$, find $f(g(x))$.

Solution: $f(g(x)) = \sqrt{\left(5x^2 - 2\right)} + 1 = \boxed{\sqrt{5x^2 - 2} + 1}$

82. Given $g(x) = \sqrt{x^2 - 5x}$ and $f(x) = x^2 + 1$, find $f(g(x))$.

Solution: $f(g(x)) = \left(\sqrt{x^2 - 5x}\right)^2 + 1 = \boxed{x^2 - 5x + 1}$

83. Given $f(x) = 3x - 2$ and $g(x) = x + 1$, find $f(g(x))$.

Solution: *Note:*

$f(g(x)) = 3\left(x + 1\right) - 2 = \boxed{3x + 1}$

84. Given $f(x) = x^2 + x^{\frac{1}{2}}$ and $g(x) = x^4$, find $f(g(x))$.

Solution: $f(g(x)) = \left(x^4\right)^2 + \left(x^4\right)^{\frac{1}{2}} = \boxed{x^8 + x^2}$

85. Given $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{2}}$ and $g(x) = x^3$, find $f(g(x))$.

Solution: $f(g(x)) = \left(x^3\right)^{\frac{1}{3}} + \left(x^3\right)^{\frac{1}{2}} = \boxed{x + x^{\frac{3}{2}}}$

How to Find the Rule of Inverse Function

- Choose an output variable and set equal to the rule of the function. (For example, $y = f(x)$.)
- Solve for the input variable. (For example, x.)
- Interchange the input variable and output variable.

86. Find the inverse function of $f(x) = \dfrac{1}{3x+2}$.

Solution:

$$f = \frac{1}{3x+2} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 3xf + 2f = 1 \underset{\text{Group terms with } x}{\Longrightarrow} 3xf = 1 - 2f$$

$$\underset{\text{Factor } x}{\Longrightarrow} x(3f) = 1 - 2f \underset{\text{Solve}}{\Longrightarrow} x = \frac{1-2f}{3f} \underset{\text{Interchange variables}}{\Longrightarrow} \boxed{f^{-1}(x) = \frac{1-2x}{3x}}$$

87. Find the inverse function of $f(s) = \dfrac{-7}{3s+2}$.

Solution:

$$f = \frac{-7}{3s+2} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 3sf + 2f = -7 \underset{\text{Group terms with } s}{\Longrightarrow} 3sf = -7 - 2f$$

$$\underset{\text{Factor } s}{\Longrightarrow} s(3f) = -7 - 2f \underset{\text{Solve}}{\Longrightarrow} s = \frac{-7-2f}{3f} \underset{\text{Interchange variables}}{\Longrightarrow} \boxed{f^{-1}(s) = \frac{-7-2s}{3s}}$$

88. Find the inverse function of $m(t) = \dfrac{2t+5}{3t}$.

Solution:

$$m = \frac{2t+5}{3t} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 3tm = 2t + 5 \underset{\text{Group terms with } t}{\Longrightarrow} 3tm - 2t = 5$$

$$\underset{\text{Factor } t}{\Longrightarrow} t(3m-2) = 5 \underset{\text{Solve}}{\Longrightarrow} t = \frac{5}{3m-2} \underset{\text{Interchange variables}}{\Longrightarrow} \boxed{m^{-1}(t) = \frac{5}{3t-2}}$$

89. Find the inverse function of $v(t) = \dfrac{7t+2}{3t-5}$.

Solution:

$$v = \frac{7t+2}{3t-5} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 3tv - 5v = 7t + 2 \underset{\text{Group terms with } t}{\Longrightarrow} 3tv - 7t = 2 + 5v$$

$$\underset{\text{Factor } t}{\Longrightarrow} t(3v-7) = 2 + 5v \underset{\text{Solve}}{\Longrightarrow} t = \frac{2+5v}{3v-7} \underset{\text{Interchange variables}}{\Longrightarrow} \boxed{v^{-1}(t) = \frac{2+5t}{3t-7}}$$

90. Find the inverse function of $g(t) = \dfrac{7}{2t-3}$.

Solution:

$$g = \frac{7}{2t-3} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 2tg - 3g = 7 \underset{\text{Group terms with } t}{\Longrightarrow} 2tg = 7 + 3g$$

$$\underset{\text{Factor } t}{\Longrightarrow} t(2g) = 7 + 3g \underset{\text{Solve}}{\Longrightarrow} t = \frac{7+3g}{2g} \underset{\text{Interchange variables}}{\Longrightarrow} \boxed{g^{-1}(t) = \frac{3t+7}{2t}}$$

91. Find the inverse function of $y(x) = \dfrac{x^3-2}{x^3+5}$.

Solution:

$$y = \frac{x^3-2}{x^3+5} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} x^3y + 5y = x^3 - 2 \underset{\text{Group terms with } x^3}{\Longrightarrow} x^3y - x^3 = -2 - 5y$$

$$\underset{\text{Factor } x^3}{\Longrightarrow} x^3(y-1) = -2 - 5y \underset{\text{Solve}}{\Longrightarrow} x^3 = \frac{-2-5y}{y-1} \Longrightarrow x = \sqrt[3]{\frac{-2-5y}{y-1}}$$

$$\underset{\text{Interchange variables}}{\Longrightarrow} \boxed{y^{-1}(x) = \sqrt[3]{\frac{-2-5x}{x-1}}}$$

92. Find the inverse function of $f(x) = \sqrt{7x+2}$.

Solution:

$$f = \sqrt{7x+2} \underset{\text{Both side to power 2}}{\Longrightarrow} f^2 = 7x+2 \underset{\text{Group all terms with } x}{\Longrightarrow} 7x = f^2 - 2$$

$$\underset{\text{Solve for } x}{\Longrightarrow} x = \frac{f^2-2}{7} \underset{\text{Interchange the variables}}{\Longrightarrow} \boxed{f^{-1}(x) = \frac{x^2-2}{7}}$$

93. Find the inverse function of $u(t) = \dfrac{5}{\sqrt{2t}}$.

Solution:

$$f = \frac{5}{\sqrt{2t}} \underset{\text{Both sides to power 2}}{\Longrightarrow} f^2 = \frac{5^2}{2t} \underset{\text{Multiply both sides by the denominator}}{\Longrightarrow} 2tf^2 = 25$$

$$\underset{\text{Solve for } t}{\Longrightarrow} t = \frac{25}{2f^2} \underset{\text{Interchange the variables}}{\Longrightarrow} \boxed{u^{-1}(t) = \frac{25}{2t^2}}$$

94. Find the inverse function of $g(y) = \sqrt{3y} + 7$.

Solution:

$$g = \sqrt{3y} + 7 \underset{\text{Isolate the radical}}{\Longrightarrow} \sqrt{3y} = g - 7 \underset{\text{Both sides to power 2}}{\Longrightarrow} 3y = (g-7)^2$$

$$\underset{\text{Solve for } y}{\Longrightarrow} y = \frac{(g-7)^2}{3} \underset{\text{Interchange the variables}}{\Longrightarrow} \boxed{g^{-1}(y) = \frac{(y-7)^2}{3}}$$

95. Find the inverse function of $u(r) = 5 + \sqrt{2r-3}$.

Solution:

$$u = 5 + \sqrt{2r-3} \underset{\text{Isolate the radical}}{\Longrightarrow} \sqrt{2r-3} = u - 5 \underset{\text{Both sides to power 2}}{\Longrightarrow} 2r - 3 = (u-5)^2$$

$$\underset{\text{Solve for } u}{\Longrightarrow} y = \frac{(u-5)^2+3}{2} \underset{\text{Interchange the variables}}{\Longrightarrow} \boxed{u^{-1}(y) = \frac{(r-5)^2+3}{2}}$$

Simplifying Rational Expression

Simplifying extra factor:

Factor both numerator and denominator.

Simplify the common factors.

Simplifying the Sum of Rational Expressions:

Make sure each expression is simplified. (Within the expression's domain.)

Find the least common denominator. This is going to be the denominator.

Multiply all rational piece to make the new numerator.

After forming the new fraction, check if it can be simplified again.

96. Simplify, within its domain, as much as possible $\dfrac{(x^2+9)(x-3)^2}{x^4-81}$.

Solution:

Within the domain $x \neq \pm 3$:

$$\frac{(x^2+9)(x-3)^2}{x^4-81} \underset{\text{Factor using the difference of squares formula } A=x^2 \text{ and } B=9.}{=} \frac{(x^2+9)(x-3)^2}{(x^2-9)(x^2+9)}$$

$$\underset{\text{Factor using the difference of squares formula } A=x \text{ and } B=3.}{=} \frac{(x^2+9)(x-3)^2}{(x-3)(x+3)(x^2+9)} \underset{\text{simplify}}{=} \boxed{\frac{(x-3)}{(x+3)}}$$

97. Simplify, within its domain, as much as possible $\dfrac{xy+2zy}{x^2+4xz+4z^2}$.

Solution:

Where $x \neq -2z$:

$$\frac{xy+2zy}{x^2+4xz+4z^2} \underset{\text{Factor the numerator.}}{=} \frac{y(x+2z)}{x^2+4xz+4z^2}$$

$$\underset{\text{Use the binomial formula to factor.}}{=} \frac{y(x+2z)}{(x+2z)^2} \underset{\text{simplify}}{=} \boxed{\frac{y}{x+2z}}$$

98. Simplify, within its domain, as much as possible $\dfrac{x^2+xy}{x^2+xy-3x-3y}$.

Solution:

Where $x \neq -y$ *and* $x \neq 3$:

$$\frac{x^2+xy}{x^2+xy-3x-3y} \underset{\text{Factor the numerator.}}{=} \frac{x(x+y)}{x^2+xy-3x-3y} \underset{\text{Factor by regrouping. } x(x+y)-3(x+y).}{=} \frac{x\cancel{(x+y)}}{\cancel{(x+y)}(x-3)} \underset{\text{simplify}}{=} \boxed{\frac{x}{x-3}}$$

99. Simplify, within its domain, as much as possible $\dfrac{2x^3-6x^2+x-3}{x-3}$.

Solution:

Within its domain $x \neq 3$:

$$\frac{2x^3-6x^2+x-3}{x-3} \underset{\text{Factor numerator by regrouping. } 2x^2(x-3)+(x-3).}{=} \frac{(2x^2+1)\cancel{(x-3)}}{\cancel{(x-3)}} \underset{\text{Simplify.}}{=} \boxed{2x^2+1}$$

100. Simplify, within its domain, as much as possible $\dfrac{x^3+5x^2+6x}{x^3-4x}$.

Solution:

Domain: $x \neq 0, x \neq \pm 2$

$$\frac{x^3+5x^2+6x}{x^3-4x} \underset{\text{Factor the } x \text{ out of numerator and denominator.}}{=} \frac{\cancel{x}(x^2+5x+6)}{\cancel{x}(x^2-4)} = \frac{x^2+5x+6}{x^2-4}$$

$$\underset{\text{Factor the numerator using quadratic formula.}}{=} \frac{(x+2)(x+3)}{x^2-4}$$

$$\underset{\text{Factor the denominator using the difference of squares. } A=x \text{ and } B=2.}{=} \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)} \underset{\text{simplify}}{=} \boxed{\frac{x+3}{x-2}}$$

101. Simplify as much as possible $\dfrac{3}{x+2}+\dfrac{3}{x-2}+\dfrac{1}{x^2-4}$.

Solution:

The least common denominator is $(x-2)(x+2) = \underbrace{x^2-4}_{\text{The denominator}}$.

The numerator is $\left(\dfrac{3}{x+2}+\dfrac{3}{x-2}+\dfrac{1}{x^2-4}\right)(x^2-4)$

Simplify:

$$= \frac{3(x^2-4)}{x+2}+\frac{3(x^2-4)}{x-2}+\frac{\cancel{x^2-4}}{\cancel{x^2-4}} = \frac{3(x-2)\cancel{(x+2)}}{\cancel{x+2}}+\frac{3\cancel{(x-2)}(x+2)}{\cancel{x-2}}+1$$

$$= 3x-(3)(2)+3x+(3)(2)+1 = \underbrace{6x+1}_{\text{The numerator}}$$

The answer is $\boxed{\dfrac{6x+1}{x^2-4}}$

102. Simplify as much as possible $\dfrac{-1}{x}+\dfrac{3}{x^2+1}+\dfrac{1}{x^3+x}$.

Solution:

The least common denominator is $x(x^2+1) = \underbrace{x^3+x}_{\text{The denominator}}$.

The numerator is $\left(\dfrac{-1}{x}+\dfrac{3}{x^2+1}+\dfrac{1}{x^3+x}\right)(x^3+x)$

Simplify:

$$= \frac{-(x^3+x)}{x}+\frac{3(x^3+x)}{x^2+1}+\frac{\cancel{x^3+x}}{\cancel{x^3+x}} = \frac{-\cancel{x}(x^2+1)}{\cancel{x}}+\frac{3x\cancel{(x^2+1)}}{\cancel{x^2+1}}+1$$

$$= -x^2-1+3x+1 = \underbrace{-x^2+3x}_{\text{The numerator}}$$

So the expression is simplifying to $\dfrac{-x^2+2x}{x^3+x}$ *but we are not done since this can be simplified.*

$$\frac{-x^2+3x}{x^3+x} = \frac{\cancel{x}(-x+3)}{\cancel{x}(x^2+1)}$$

The answer is $\boxed{\dfrac{-x+3}{x^2+1}}$

103. Simplify as much as possible $\sqrt{3x-1} - \dfrac{x+4}{\sqrt{3x-1}}$.

Solution:

The least common denominator is $\underbrace{\sqrt{3x-1}}_{\text{The denominator}}$.

The numerator is $\left(\sqrt{3x-1} - \dfrac{x+4}{\sqrt{3x-1}}\right)(\sqrt{3x-1})$

Simplify:

$$3x-1-\frac{(x+4)\cancel{(\sqrt{3x-1})}}{\cancel{(\sqrt{3x-1})}} = 3x-1-(x+4) = 3x-1-x-4 = \underbrace{2x-5}_{\text{The numerator}}$$

The answer is $\boxed{\dfrac{2x-5}{\sqrt{3x-1}}}$

104. Simplify as much as possible $\dfrac{x}{x+y} - \dfrac{y}{x}$.

Solution:

The least common denominator is $x(x+y) = \underbrace{x^2+xy}_{\text{The denominator}}$.

The numerator is $\left(\dfrac{x}{x+y} - \dfrac{y}{x}\right)(x(x+y))$

Simplify:

$$= \frac{x(x\cancel{(x+y)})}{\cancel{(x+y)}} - \frac{y(\cancel{x}(x+y))}{\cancel{x}} = x^2 - y(x+y) = x^2 - xy - y^2 = \underbrace{x^2-xy-y^2}_{\text{The numerator}}$$

The answer is $\boxed{\dfrac{x^2-xy-y^2}{x^2+xy}}$

105. Simplify as much as possible $\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}$.

Solution:

The least common denominator is $\underbrace{(x+1)(x+h+1)}_{\text{The denominator}}$.

The numerator is $\left(\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}\right)(x+1)(x+h+1)$

Simplify:

$= \dfrac{(x+h)(x+1)\cancel{(x+h+1)}}{\cancel{x+h+1}} - \dfrac{x\cancel{(x+1)}(x+h+1)}{\cancel{x+1}} = (x+h)(x+1) - x(x+h+1) =$
$x^2 + hx + x + h - x^2 - xh - x = \underbrace{h}_{\text{The numerator}}$

The answer is $\boxed{\dfrac{h}{(x+1)(x+h+1)}}$